Section 1

Analog Communications

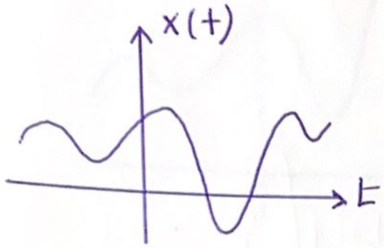
(SSP)

Sheet 1 - Solutions

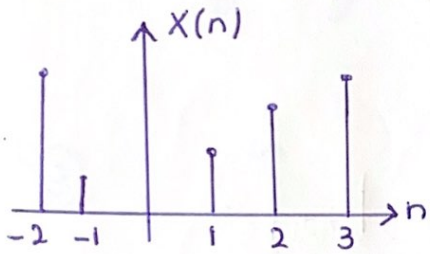
Signals Classification according to

Time

→ Continous-time

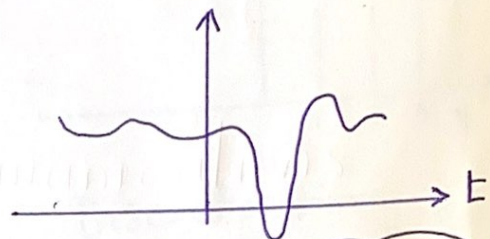


→ Discrete-time



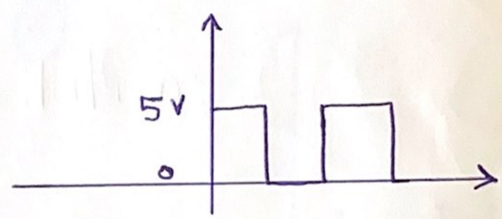
Amplitude.

→ Analog - Signal.



Continous Amplitude

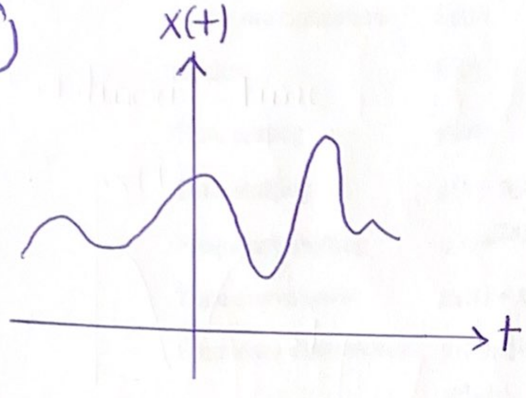
→ Digital Signal



Discrete Amplitude

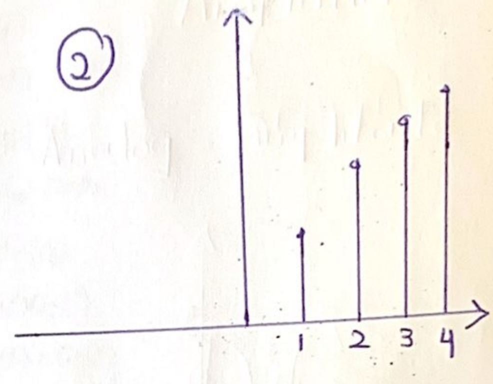
• Classify the following signals according to time and amplitude.

1



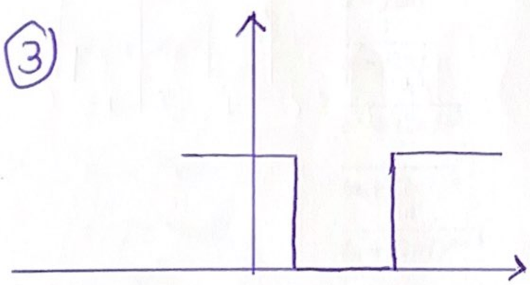
Continous time
Analog Signal

2



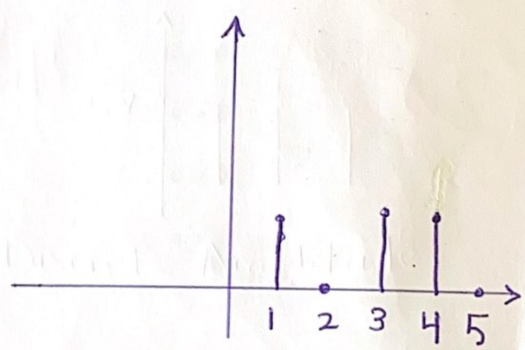
Discrete time.
Analog Signal

3



Continous time
Digital Signal

4



Discrete time
Digital signal

Properties of Fourier Transform Operations

Operation	$g(t)$	$G(f)$
Superposition	$g_1(t) + g_2(t)$	$G_1(f) + G_2(f)$
Scalar multiplication	$kg(t)$	$kG(f)$
Duality	$G(t)$	$g(-f)$
Time scaling	$g(at)$	$\frac{1}{\|a\|} G\left(\frac{f}{a}\right)$
Time shifting	$g(t - t_0)$	$G(f)e^{-j2\pi f t_0}$
Frequency shifting	$g(t)e^{j2\pi f_0 t}$	$G(f - f_0)$
Time convolution	$g_1(t) * g_2(t)$	$G_1(f)G_2(f)$
Frequency convolution	$g_1(t)g_2(t)$	$G_1(f) * G_2(f)$
Time differentiation	$\frac{d^n g(t)}{dt^n}$	$(j2\pi f)^n G(f)$
Time integration	$\int_{-\infty}^{t} g(x)\,dx$	$\frac{G(f)}{j2\pi f} + \frac{1}{2}G(0)\delta(f)$

Short Table of Fourier Transforms

	$g(t)$	$G(f)$	
1	$e^{-at}u(t)$	$\frac{1}{a + j2\pi f}$	$a > 0$
2	$e^{at}u(-t)$	$\frac{1}{a - j2\pi f}$	$a > 0$
3	$e^{-a\|t\|}$	$\frac{2a}{a^2 + (2\pi f)^2}$	$a > 0$
4	$te^{-at}u(t)$	$\frac{1}{(a + j2\pi f)^2}$	$a > 0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a + j2\pi f)^{n+1}}$	$a > 0$
6	$\delta(t)$	1	
7	1	$\delta(f)$	
8	$e^{j2\pi f_0 t}$	$\delta(f - f_0)$	
9	$\cos 2\pi f_0 t$	$0.5\,[\delta(f + f_0) + \delta(f - f_0)]$	
10	$\sin 2\pi f_0 t$	$j0.5\,[\delta(f + f_0) - \delta(f - f_0)]$	
11	$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$	
12	$\operatorname{sgn} t$	$\frac{2}{j2\pi f}$	
13	$\cos 2\pi f_0 t\, u(t)$	$\frac{1}{4}[\delta(f - f_0) + \delta(f + f_0)] + \frac{j2\pi f}{(2\pi f_0)^2 - (2\pi f)^2}$	
14	$\sin 2\pi f_0 t\, u(t)$	$\frac{1}{4j}[\delta(f - f_0) - \delta(f + f_0)] + \frac{2\pi f_0}{(2\pi f_0)^2 - (2\pi f)^2}$	
15	$e^{-at}\sin 2\pi f_0 t\, u(t)$	$\frac{2\pi f_0}{(a + j2\pi f)^2 + 4\pi^2 f_0^2}$	$a > 0$
16	$e^{-at}\cos 2\pi f_0 t\, u(t)$	$\frac{a + j2\pi f}{(a + j2\pi f)^2 + 4\pi^2 f_0^2}$	$a > 0$

Recall (Fourier)

Fourier Series (FS)

- Applied on periodic signals to transform from time domain to frequency domain

$$x(f) = \sum_{n=-\infty}^{\infty} D_n \, e^{jn\omega_0 t}$$

$$D_n = \frac{1}{T_0} \int_{\langle T_0 \rangle} x(t)\, e^{-jn\omega_0 t} \, dt$$

express the periodic signal $x(t)$ interms of coefficient D_n, where T_0 is the period. (Discrete)

Fourier Transform (FT)

- Applied on Aperiodic Signals

$$\underset{\text{T.D}}{x(t)} \longrightarrow \underset{\text{F.D}}{X(f)}$$

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t} \, dt$$

check tables of F.T

(continous).

Some properties of F.T:

① $f(t-\alpha) \xrightarrow{FT} G(F)\, e^{-j2\pi\alpha f}$ Same sign.

② $g(t)\, e^{+j2\pi\alpha t} \xrightarrow{FT} G(f-\alpha)$ different sign.

③ $\frac{d}{dt} g(t) \xrightarrow{FT} j2\pi f\, G(f)$.

④ $g_1(t) \circledast g_2(t) \xrightarrow{FT} G_1(f) \cdot G_2(f)$

⑤ Duality

$$\begin{tikzcd} g(t) \arrow[r] \arrow[dr] & G(f) \arrow[dl] \\ G(t) & g(-f) \end{tikzcd}$$

Ex. on duality:

$$F.T\left\{A \operatorname{rect}\left(\frac{t}{\tau}\right)\right\} \xrightarrow[\text{table}]{\text{from F.T}} A\tau \operatorname{sinc}(f\tau).$$

find $F.T\{\operatorname{sinc}(t.\tau)\}$ we can't find F.T of Sinc in the tables so use duality property.

$$\operatorname{rect}\left(\frac{t}{\tau}\right) \longrightarrow \tau \operatorname{sinc}(f\tau)$$

$$\begin{tikzcd} \frac{1}{\tau}\operatorname{rect}\left(\frac{t}{\tau}\right) \arrow[r] \arrow[dr] & \operatorname{sinc}(f\tau) \arrow[dl] \\ \operatorname{sinc}(t\tau) & \frac{1}{\tau}\operatorname{rect}\left(\frac{-f}{\tau}\right) \end{tikzcd}$$

rect is an even fn.

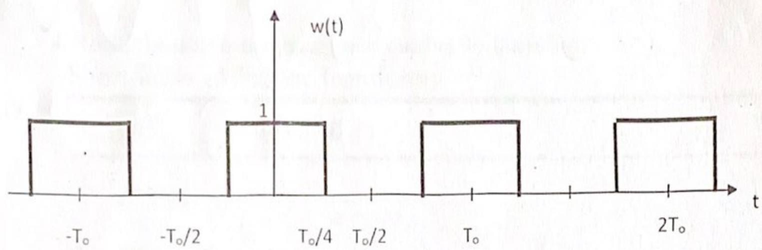
$$\therefore \operatorname{rect}\left(\frac{-f}{\tau}\right) = \operatorname{rect}\left(\frac{f}{\tau}\right)$$

$$\therefore F.T\{\operatorname{sinc}(t\tau)\} = \frac{1}{\tau}\operatorname{rect}\left(\frac{f}{\tau}\right).$$

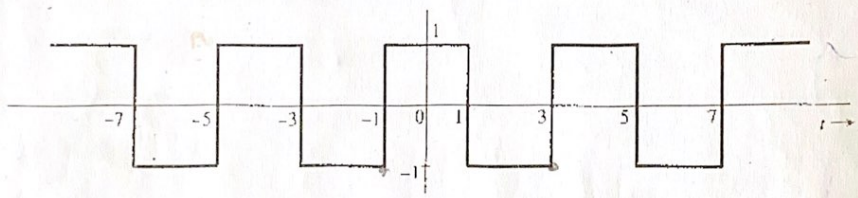
Tutorial 1

Signals - Review

1. Find the exponential Fourier series for the periodic square wave shown in the following figure.



2. For the following periodic signal, find the exponential Fourier series, then sketch the amplitude and phase spectra.



3. Find the Fourier transform for the signal $g(t) = e^{-at}\, u(t)$, $a>0$.

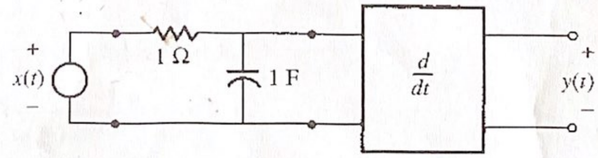
4. Verify Parseval's theorem for the signal $g(t) = e^{-at}u(t)$ $(a > 0)$.

5. Estimate the essential bandwidth W (in rad/s) of the signal $e^{-at}u(t)$ if the essential band is required to contain 95% of the signal energy.

6. Find the power of the output voltage y(t) of the system shown in the following figure if the input voltage PSD

$S_x(f) = \Pi(\pi f)$.

Calculate the power of the input signal x(t).



7. Sketch the following functions:

a) Rect(t/2)
b) Tri(3ω/100)
c) Rect(t-10/8)
d) Sinc(πω/5)

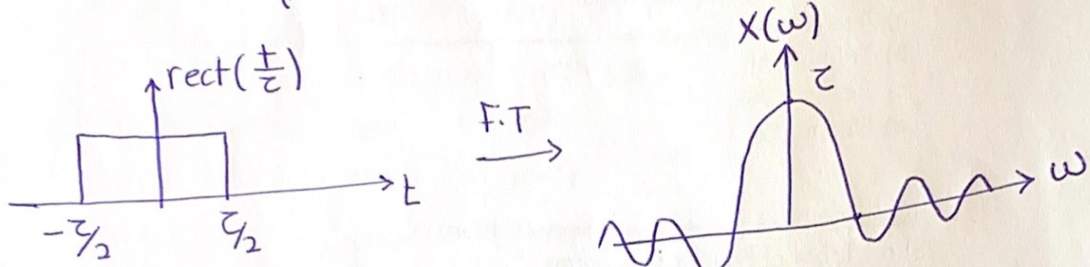
8. Recall the table below, then prove number 9, 10 and 13.

Short Table of Fourier Transforms

	$g(t)$	$G(f)$	
1	$e^{-at}u(t)$	$\frac{1}{a+j2\pi f}$	$a>0$
2	$e^{at}u(-t)$	$\frac{1}{a-j2\pi f}$	$a>0$
3	$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+(2\pi f)^2}$	$a>0$
4	$te^{-at}u(t)$	$\frac{1}{(a+j2\pi f)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\frac{n!}{(a+j2\pi f)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$\delta(f)$	
8	$e^{j2\pi f_0 t}$	$\delta(f-f_0)$	
9	$\cos 2\pi f_0 t$	$0.5\,[\delta(f+f_0)+\delta(f-f_0)]$	
10	$\sin 2\pi f_0 t$	$j0.5\,[\delta(f+f_0)-\delta(f-f_0)]$	
11	$u(t)$	$\frac{1}{2}\delta(f)+\frac{1}{j2\pi f}$	
12	$\operatorname{sgn} t$	$\frac{2}{j2\pi f}$	
13	$\cos 2\pi f_0 t\, u(t)$	$\frac{1}{4}[\delta(f-f_0)+\delta(f+f_0)]+\frac{j2\pi f}{(2\pi f_0)^2-(2\pi f)^2}$	
14	$\sin 2\pi f_0 t\, u(t)$	$\frac{1}{4j}[\delta(f-f_0)-\delta(f+f_0)]+\frac{2\pi f_0}{(2\pi f_0)^2-(2\pi f)^2}$	
15	$e^{-at}\sin 2\pi f_0 t\, u(t)$	$\frac{2\pi f_0}{(a+j2\pi f)^2+4\pi^2 f_0^2}$	$a>0$
16	$e^{-at}\cos 2\pi f_0 t\, u(t)$	$\frac{a+j2\pi f}{(a+j2\pi f)^2+4\pi^2 f_0^2}$	$a>0$

Important:

① F.T of $\left\{ rect\left(\frac{t}{\tau}\right) \right\} \longrightarrow \tau \, sinc\left(\frac{\omega\tau}{2}\right)$.



Nulls @ $\frac{\omega\tau}{2} = \pm n\pi$.

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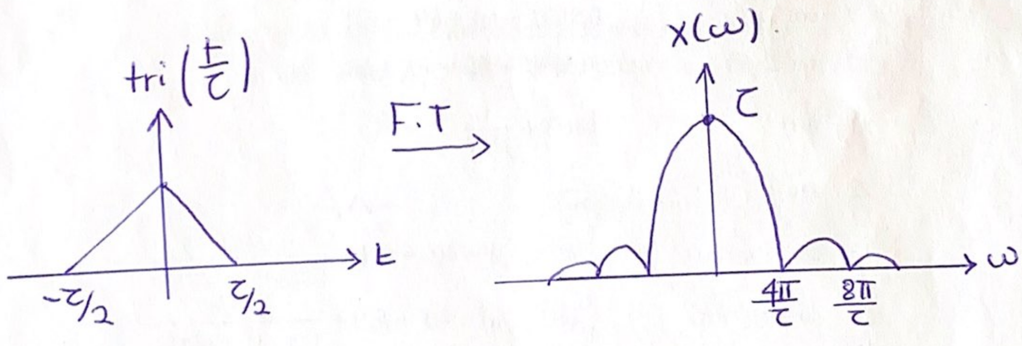
② F.T$\left\{ \Delta\left(\frac{t}{\tau}\right) \right\} \longrightarrow \frac{\tau}{2} \, sinc^2\left(\frac{\omega\tau}{4}\right)$



Nulls @ $\frac{\omega\tau}{4} = \pm n\pi$.

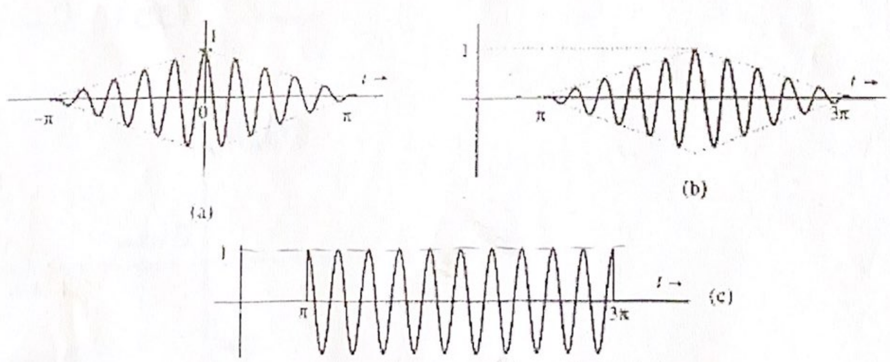
Recall the Fourier transform properties in the table below, then answer questions 9 and 10.

Properties of Fourier Transform Operations

Operation	$g(t)$	$G(f)$
Superposition	$g_1(t) + g_2(t)$	$G_1(f) + G_2(f)$
Scalar multiplication	$kg(t)$	$kG(f)$
Duality	$G(t)$	$g(-f)$
Time scaling	$g(at)$	$\frac{1}{\lvert a\rvert} G\left(\frac{f}{a}\right)$
Time shifting	$g(t - t_0)$	$G(f)e^{-j2\pi f t_0}$
Frequency shifting	$g(t)e^{j2\pi f_0 t}$	$G(f - f_0)$
Time convolution	$g_1(t) * g_2(t)$	$G_1(f)G_2(f)$
Frequency convolution	$g_1(t)g_2(t)$	$G_1(f) * G_2(f)$
Time differentiation	$\frac{d^n g(t)}{dt^n}$	$(j2\pi f)^n G(f)$
Time integration	$\int_{-\infty}^{t} g(x)\, dx$	$\frac{G(f)}{j2\pi f} + \frac{1}{2}G(0)\delta(f)$

9. Apply the duality property to show that:
   a) $0.5[\delta(t) + (j/\pi t)] \leftrightarrow u(f)$
   b) $\delta(t+T) + \delta(t-T) \leftrightarrow 2\cos(2\pi fT)$

10. The signals in the following figure are modulated with carrier cos10t, Find the Fourier transforms of these signals using suitable Fourier properties, then sketch the spectra.



Hint:

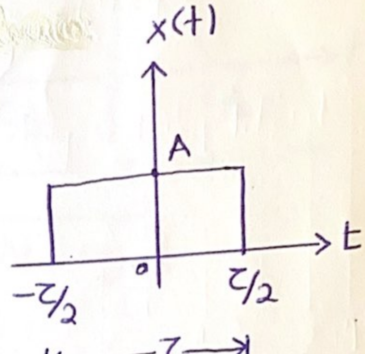
$$\Delta\left(\frac{t}{2\pi}\right) \Longleftrightarrow \pi\, \mathrm{sinc}^2\left(\frac{\pi\omega}{2}\right)$$

$$\mathrm{rect}\left(\frac{t}{2\pi}\right) \Longleftrightarrow 2\pi\, \mathrm{sinc}(\pi\omega)$$

Recall some signals: (functions)

① Rect. function:

$$X(t) = A \ \mathrm{rect}\left(\frac{t}{\tau}\right),$$



Where, A is the amplitude.

τ is the width.

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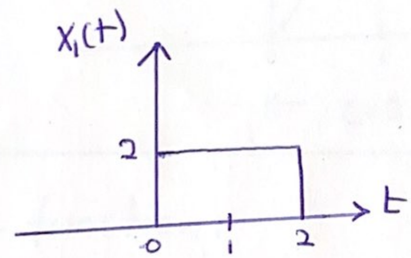
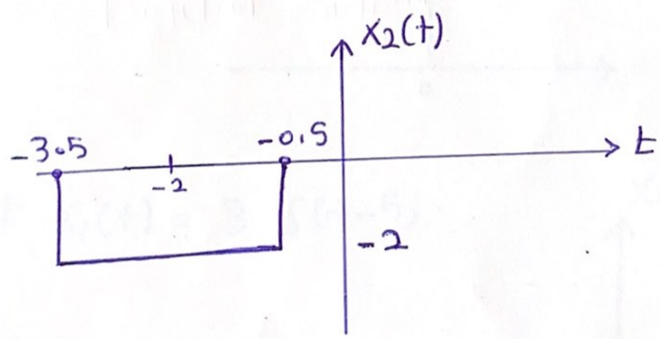
if $x_1(t) = 2 \ \mathrm{rect}\left(\frac{t-1}{2}\right)$ → shift right by 1.



$X_2(t) = -2 \ \mathrm{rect}\left(\frac{t+2}{3}\right)$. → shift left by 2.



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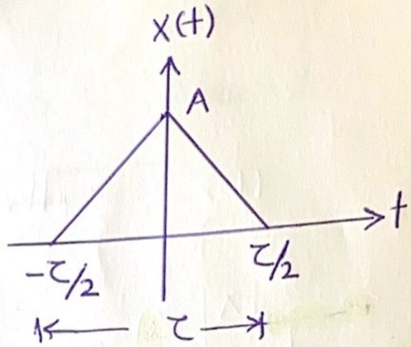
2 Tri - function:

$x(t) = A \, tri\left(\frac{t}{\tau}\right)$,



Where, A is the amplitude

τ is the half width

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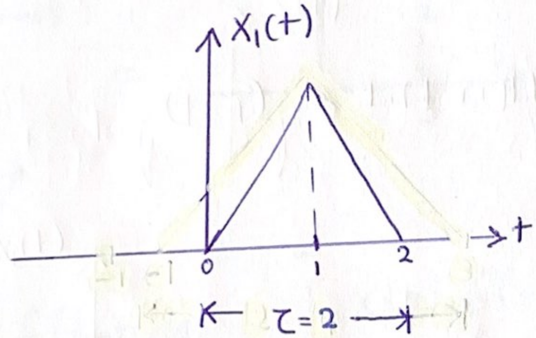
if $x_1(t) = 4 \; tri\left(\frac{t-1}{2}\right)$ → shift right by 1



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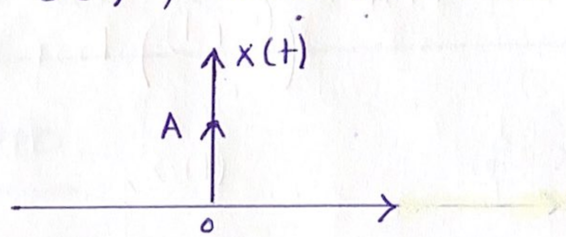
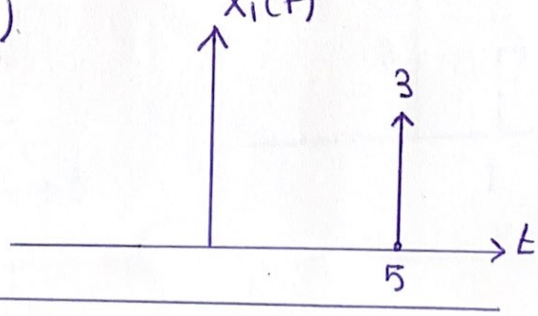
3 Delta- function:

$x(t) = A \; \delta(t)$, where A is the amplitude.

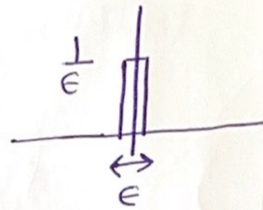


if $x_1(t) = 3 \; \delta(t-5)$.



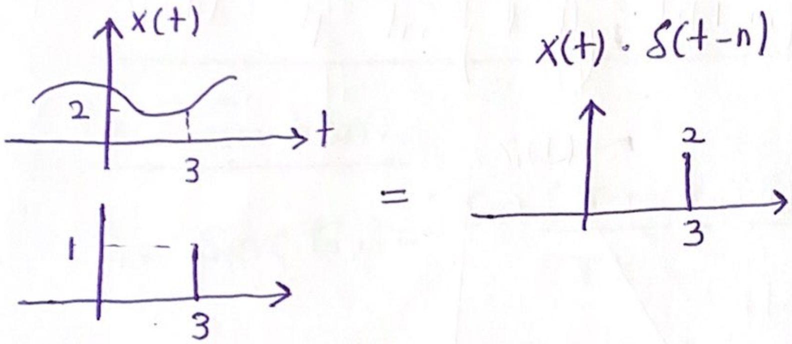
Properties of delta function:

1. $\int_{-\infty}^{\infty} \delta(t)\, dt = 1.$ $\Longrightarrow$



$\epsilon <<$
Area = 1.

2. $x(t) \cdot \delta(t-n) = x(n)\, \delta(t-n).$

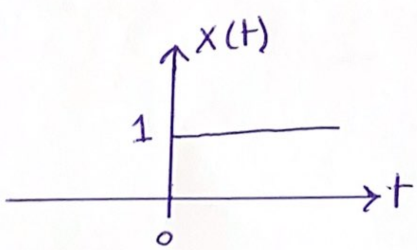


3. $\int_{-\infty}^{\infty} x(t) \cdot \delta(t-n)\, dt = \int_{-\infty}^{\infty} x(n)\, \delta(t-n)\, dt = x(n) \cdot 1$

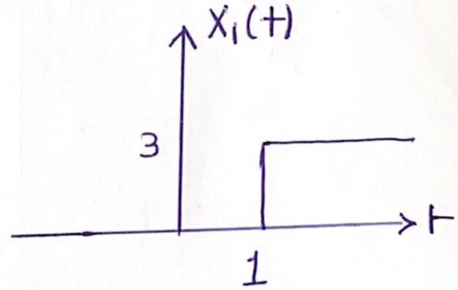
4. $x(t) \circledast \delta(t-n) = x(t-n)$

4. Unit step:

$x(t) = u(t)$

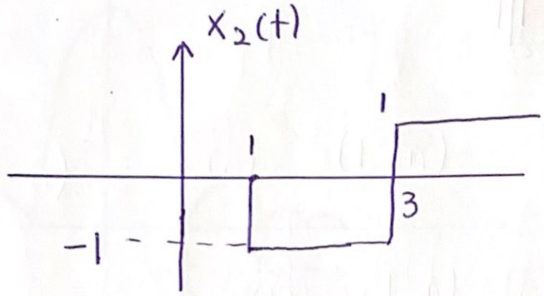


if $x_1(t) = 3\, u(t-1)$



if $x_2(t) = 2\,u(t-3) - u(t-1)$

$\therefore\ x_2(t) = -u(t-1) + 2\,u(t-3)$



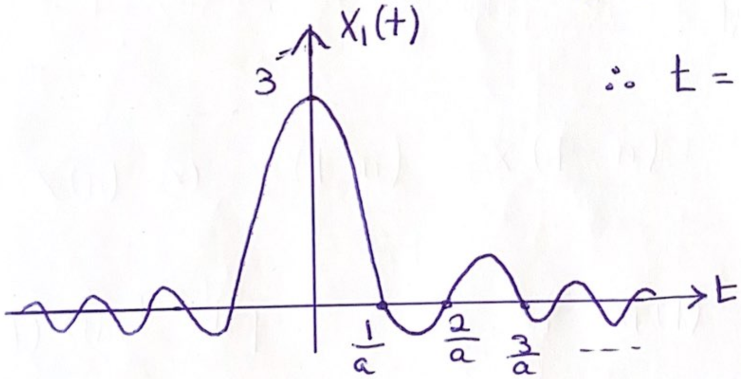
⑤ Sinc function:

$$x(t) = \text{Sinc}(t) = \frac{\sin(\pi t)}{\pi t} = \frac{\sin(t)}{t}$$

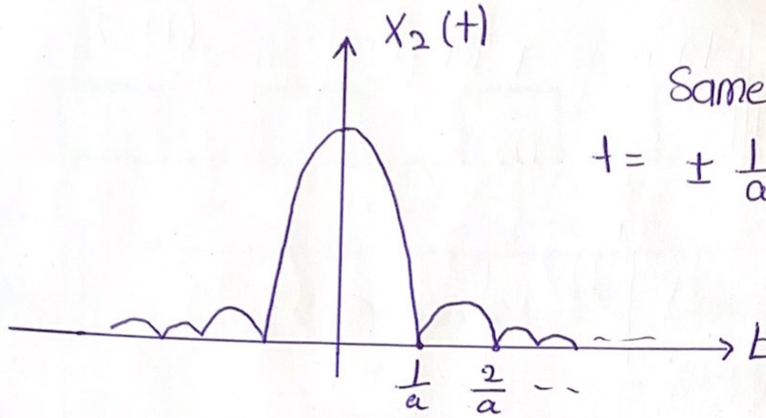
if $x_1(t) = 3\ \text{sinc}(at)$ $\rightarrow z = at$

Nulls @ $z = at = \pm 1, \pm 2, \pm 3, \ldots$

$\therefore\ t = \pm \frac{1}{a}, \pm \frac{2}{a}, \pm \frac{3}{a}, \ldots$



$X_2(t) = \text{Sinc}^2(at)$



Same Nulls

$t = \pm\frac{1}{a}, \pm\frac{2}{a}, \dots$

Note:

$$\frac{\sin \pi t}{t} = Sa(t)$$ Sampling function.

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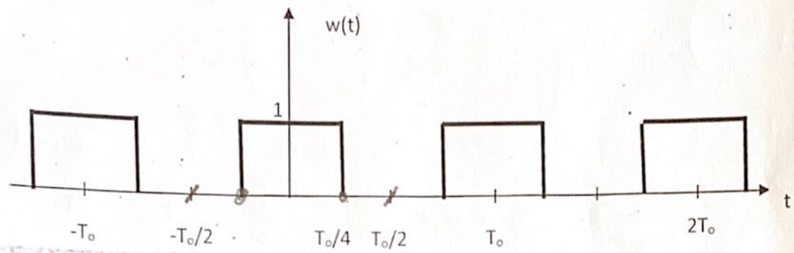
1. Find the exponential Fourier series for the periodic square wave shown in the following figure.



Solution: Find exponential F.S (because periodic signal with period T_0).

$$x(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

$$D_n = \frac{1}{T_0} \int_{\langle T_0 \rangle} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$\therefore D_n = \frac{1}{T_0} \int_{-\frac{T_0}{4}}^{\frac{T_0}{4}} 1 \cdot e^{-jn\omega_0 t}\, dt$$

$$D_n = \frac{1}{-jn\omega_0 T_0} \left[e^{-jn\omega_0 \frac{T_0}{4}} - e^{jn\omega_0 \frac{T_0}{4}} \right].$$

recall $\Rightarrow$ $\cos(x) = \frac{1}{2}\left[e^{jx} + e^{-jx} \right]$

$$\sin(x) = \frac{1}{2j}\left[e^{jx} - e^{-jx} \right].$$

$$\therefore D_n = \frac{-2j}{-jn\omega_0 T_0} \sin\left(\frac{n\omega_0 T_0}{4}\right) \Rightarrow \omega_0 = \frac{2\pi}{T_0}$$

$$D_n = \frac{\sin\left(\frac{n\pi}{2}\right)}{\frac{n\pi}{2} * 2} = \frac{1}{2}\,\text{Sinc}\left(\frac{n\pi}{2}\right)$$

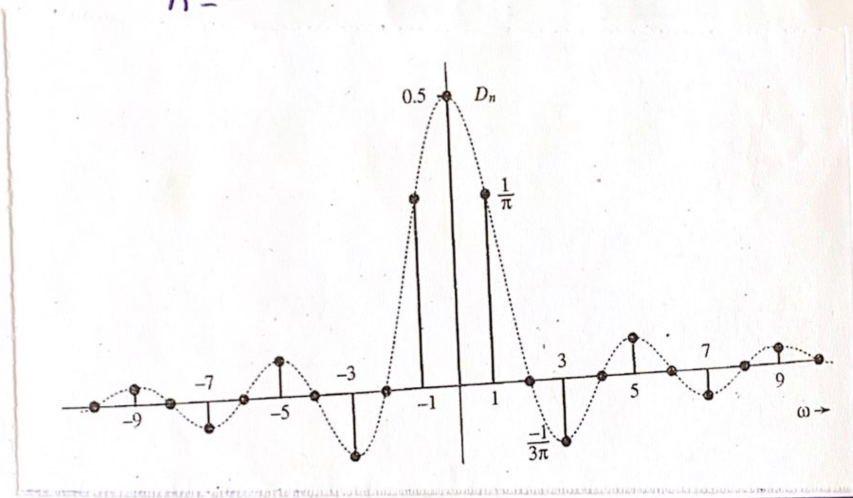
nulls @ $\frac{n\pi}{2} = \pm n\pi$

$n/2 = \pm 1, 2, 3, \ldots$

$n = \pm 2, 4, 6, \ldots$

$\therefore D_n = \frac{1}{\pi n} \cdot \sin\left(\frac{n\pi}{2}\right)$ at $n = 1, 2, 3, \ldots$

$\therefore x(t) = \sum_{n=-\infty}^{\infty} D_n \, e^{jn\omega_0 t}$ (Discrete).



at $n=0$.

$\lim_{n\to\infty} \frac{\cos^{n\pi}/_2}{\pi \cdot \frac{2}{\pi}}$

$= \frac{1}{2}$

3. Find the Fourier transform for the signal $g(t) = e^{-at}\,u(t)$, $a>0$.

Solution: $G(f) = \int_{-\infty}^{\infty} e^{-at}\, u(t)\, e^{-j2\pi f t}\, dt$ (F.T def)

$\therefore G(f) = \int_{0}^{\infty} e^{-(a+j2\pi f)t}\, dt = \frac{-1}{a+j2\pi f} \cdot e^{-(a+j2\pi f)t} \Big|_0^{\infty}$

0 ↳ due to u(t).

($\omega = 2\pi f$)

$\therefore G(\omega) = \frac{1}{a+j\omega}$, $a>0$. (or from F.T tables) directly.

$G(f) = \frac{1}{a+j2\pi f}$, $a>0$. → From fourier transform tables.

4. Verify Parseval's theorem for the signal $g(t) = e^{-at}u(t)$ $(a > 0)$.

Solution:

$E_{\text{time domain}} = E_{\text{freq. Domain}}$

to verify Parseval's theorem.

$$E_{time} = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{0}^{\infty} e^{-2at} \, dt$$

$$E_{T.D} = \frac{-1}{2a} \left[e^{-2at} \right]_0^{\infty} = \frac{-1}{2a}(-1) = \frac{1}{2a} \rightarrow 1$$

$$E_{F.D} = \int_{-\infty}^{\infty} |X(f)|^2 \, df$$

$$\therefore |X(f)| = \frac{1}{\sqrt{a^2 + (2\pi f)^2}}$$

$$\therefore E_{F.D} = \int_{-\infty}^{\infty} \frac{1}{a^2 + (2\pi f)^2} \, df = \frac{1}{2\pi a} \left[\tan^{-1}\left(\frac{2\pi f}{a}\right) \right]_{-\infty}^{\infty}$$

$$E_{F.D} = \frac{\pi}{2\pi a} = \frac{1}{2a} \rightarrow 2$$

From 1 and 2

$$\therefore E_{T.D} = E_{F.D}$$

which verifies Parseval's theorem.

5. Estimate the essential bandwidth W (in rad/s) of the signal $e^{-at}u(t)$ if the essential band is required to contain 95% of the signal energy.

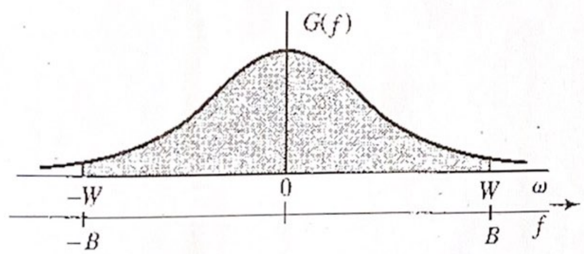
Solution :

In this case,

$$G(f) = \frac{1}{j2\pi f + a}$$

and the ESD is

$$|G(f)^2| = \frac{1}{(2\pi f)^2 + a^2}$$



This ESD is shown in Fig. 3.33. Moreover, the signal energy E_g is the area under this ESD, which has already been found to be $1/2a$. Let W rad/s be the essential bandwidth, which contains 95% of the total signal energy E_g. This means $1/2\pi$ times the shaded area in Fig. 3.33 is $0.95/2a$, that is,

$$\frac{0.95}{2a} = \int_{-W/2\pi}^{W/2\pi} \frac{df}{(2\pi f)^2 + a^2}$$

$$= \frac{1}{2\pi a} \tan^{-1} \frac{2\pi f}{a} \bigg|_{-W/2\pi}^{W/2\pi} = \frac{1}{\pi a} \tan^{-1} \frac{W}{a}$$

or

$$\frac{0.95\pi}{2} = \tan^{-1} \frac{W}{a} \Longrightarrow W = 12.7\, a \text{ rad/s}$$

In terms of hertz, the essential bandwidth is

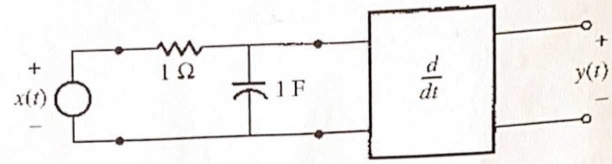
$$B = \frac{W}{2\pi} = 2.02\, a \quad \text{Hz}$$

This means that in the band from 0 (dc) to $12.7 \times a$ rad/s ($2.02 \times a$ Hz), the spectral components of $g(t)$ contribute 95% of the total signal energy; all the remaining spectral components (in the band from $2.02 \times a$ Hz to ∞) contribute only 5% of the signal energy.*

6. Find the power of the output voltage y(t) of the system shown in the following figure if the input voltage PSD

$S_X(f) = \Pi(\pi f)$.

Calculate the power of the input signal x(t).



Solution :

The ideal differentiator transfer function is $j\omega$. Hence, the transfer function of the entire sys

$$H(\omega) = \left(\frac{1}{j\omega + 1}\right)(j\omega) = \frac{j\omega}{j\omega + 1} \quad \text{and} \quad |H(\omega)|^2 = \frac{\omega^2}{\omega^2 + 1}$$

$$P_x = \int_{-1/2\pi}^{1/2\pi} \Pi(\pi f)\, df = \int_{-1/2\pi}^{1/2\pi} df = \frac{1}{\pi}$$

$$Y(f) = H(f)X(f)$$

$$|Y(f)|^2 = \frac{(2\pi f)^2}{(2\pi f)^2 + 1} \Pi(\pi f)$$

$$P_y = \int_{-1/2\pi}^{1/2\pi} \frac{(2\pi f)^2}{(2\pi f)^2 + 1} df$$

$$= \frac{1}{2\pi} \int_{-1}^{1} \frac{(\omega)^2}{(\omega)^2 + 1} d\omega$$

$$= \frac{1}{2\pi} \int_{-1}^{1} \frac{1 + (\omega)^2 - 1}{(\omega)^2 + 1} d\omega$$

$$= \frac{1}{2\pi} [\omega - \tan^{-1}\omega]_{-1}^{1}$$

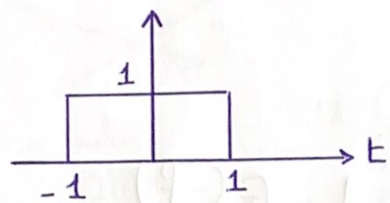
$$= \frac{1}{\pi}\left(1 - \frac{\pi}{4}\right)$$

7. Sketch the following functions:
   a) Rect(t/2)
   b) Tri(3ω/100)
   c) Rect(t-10/8)
   d) Sinc(πω/5)

Solution:

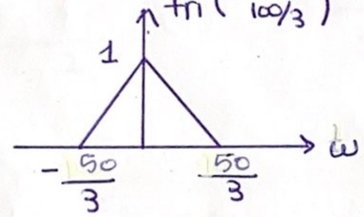
$\text{Rect}\left(\frac{t}{2}\right)$

a -



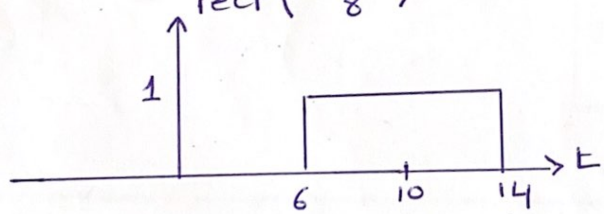
$\text{tri}\left(\frac{\omega}{100/3}\right)$

b -



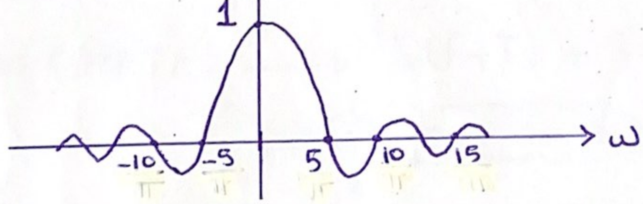
$\text{rect}\left(\frac{t-10}{8}\right)$

c -



$\text{Sinc}\left(\frac{\pi\omega}{5}\right)$

d -



Nulls @ $z = \frac{\pi\omega}{5} = \pm\pi, \pm 2\pi \pm 3\pi, \ldots, \pm n\pi$

$\omega = \pm\frac{5}{\pi}, \pm\frac{10}{\pi}, \pm\frac{15}{\pi}, \ldots$

Time Scaling property:

$$g(at) = \frac{1}{|a|} G(f/a)$$

$$a = -1$$

$$F.T\{ u(-t) \} = 1 \cdot G(-f)$$

9. Apply the duality property to show that:

a) $0.5[\delta(t) + (j/\pi t)] \leftrightarrow u(f)$

b) $\delta(t+T) + \delta(t-T) \leftrightarrow 2\cos(2\pi fT)$

Solution:

We need to have $u(f)$ so use $u(-t)$

a-

$u(-t) \qquad \frac{1}{2}\delta(f) - \frac{1}{j2\pi f}$

$\frac{1}{2}\delta(t) - \frac{1}{j2\pi t} \qquad u(f)$

$\therefore u(f) \xrightarrow{F.T} \frac{1}{2}\delta(t) + \frac{j}{2\pi t}$

$\therefore u(f) \longrightarrow \frac{1}{2}\left[\delta(t) + \frac{j}{\pi t}\right].$

① time Scaling ② Duality

-1

$F.T(u(-t)) \longrightarrow$

$1 \cdot G(-f) =$

$\frac{1}{2}\delta(-f) - \frac{1}{j2\pi f}$

but $\delta(f)$ is an even fn.

$\therefore \delta(f) = \delta(-f).$

b- $\cos(2\pi T t) \longrightarrow \frac{1}{2}[\delta(f+T) + \delta(f-T)]$

$2\cos(2\pi T t) \longrightarrow \delta(f+T) + \delta(f-T)$

replace $f \to t$

$[\delta(t+T) + \delta(t-T)] \qquad 2\cos(2\pi T(-f))$

but Cosine is an even fn.

$\cos(f) = \cos(-f)$

$\therefore 2\cos(2\pi T f) \xrightarrow{F.T} \delta(t+T) + \delta(t-T).$