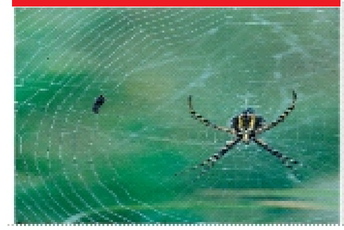
For your information



A spider detects its prey due to vibration produced in the web.

A body is said to be vibrating if it moves back and forth or to and fro about a point. Another term for vibration is oscillation. A special kind of vibratory or oscillatory motion is called the simple harmonic motion (SHM), which is the main focus of this chapter. We will discuss important characteristics of SHM and systems executing SHM. We will also introduce different types of waves and will demonstrate their properties with the help of ripple tank.

10.1 SIMPLE HARMONIC MOTION (SHM)

In the following sections we will discuss simple harmonic motion of different systems. The motion of mass attached to a spring on a horizontal frictionless surface, the motion of a ball placed in a bowl and the motion of a bob attached to a string are examples of SHM.

MOTION OF MASS ATTACHED TO A SPRING

One of the simplest types of oscillatory motion is that of horizontal mass-spring system (Fig.10.1). If the spring is stretched or compressed through a small displacement x from its mean position, it exerts a force F on the mass. According to Hooke's law this force is directly proportional to the change in length x of the spring i.e.,

$$F = -kx \quad \text{........} \quad (10.1)$$

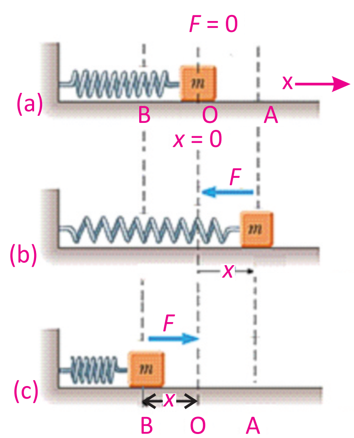


Fig.10.1: SHM of a mass-spring system

where x is the displacement of the mass from its mean position O, and k is a constant called the **spring constant** defined as

$$k = -\frac{F}{x}$$

The value of k is a measure of the stiffness of the spring. Stiff springs have large value of k and soft springs have small value of k.

As $\quad F = ma$

Therefore, $\quad k = -\frac{ma}{x}$

or $\quad a = -\frac{k}{m}x$

$$a \ \surd\ -\ x \quad \text{........} \quad (10.2)$$

It means that the acceleration of a mass attached to a spring is directly proportional to its displacement from the mean position. Hence, the horizontal motion of a mass-spring system is an example of simple harmonic motion.

The negative sign in Eq. 10.1 means that the force exerted by the spring is always directed opposite to the displacement of the mass. Because the spring force always acts towards the mean position, it is sometimes called a restoring force.

A restoring force always pushes or pulls the object performing oscillatory motion towards the mean position.

Initially the mass *m* is at rest in mean position O and the resultant force on the mass is zero (Fig.10.1-a). Suppose the mass is pulled through a distance *x* up to extreme position A and then released (Fig.10.1-b). The restoring force exerted by the spring on the mass will pull it towards the mean position O. Due to the restoring force the mass moves back, towards the mean position O. The magnitude of the restoring force decreases with the distance from the mean position and becomes zero at O. However, the mass gains speed as it moves towards the mean position and its speed becomes maximum at O. Due to inertia the mass does not stop at the mean position O but continues its motion and reaches the extreme position B.

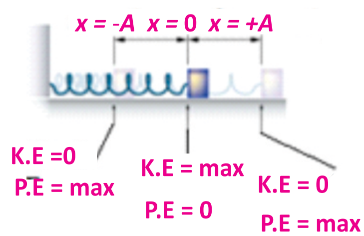
As the mass moves from the mean position O to the extreme position B, the restoring force acting on it towards the mean position steadily increases in strength. Hence the speed of the mass decreases as it moves towards the extreme position B. The mass finally comes briefly to rest at the extreme position B (Fig. 10.1-c). Ultimately the mass returns to the mean position due to the restoring force.

This process is repeated, and the mass continues to oscillate back and forth about the mean position O. Such motion of a mass attached to a spring on a horizontal frictionless surface is known as Simple Harmonic Motion (SHM).

The time period *T* of the simple harmonic motion of a mass '*m*' attached to a spring is given by the following equation:

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{......... (10.3)}$$

For your information



Kinetic and potential energy at different positions in a mass–spring system.

Tidbits

A human eardrum can oscillate back and forth up to 20,000 times in one second.

Quick Quiz

What is the displacement of an object in SHM when the kinetic and potential energies are equal?

BALL AND BOWL SYSTEM

The motion of a ball placed in a bowl is another example of simple harmonic motion (Fig 10.2). When the ball is at the mean position O, that is, at the centre of the bowl, net force acting on the ball is zero. In this position, weight of the ball acts downward and is equal to the upward normal force of the surface of the bowl. Hence there is no motion. Now if we bring the ball to position A and then release it, the ball will start moving towards the mean position O due to the restoring force caused by its weight. At position O the ball gets maximum speed and due to inertia it moves towards the extreme position B. While going towards the position B, the speed of the ball decreases due to the restoring force which acts towards the mean position. At the position B, the ball stops for a while and then again moves towards the mean position O under the action of the restoring force. This to and fro motion of the ball continues about the mean position O till all its energy is lost due to friction. Thus the to and fro motion of the ball about a mean position placed in a bowl is an example of simple harmonic motion.

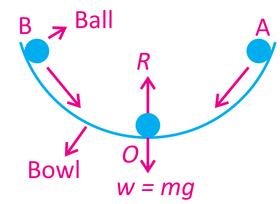


Fig. 10.2: When a ball is gently displaced from the centre of a bowl it starts oscillating about the centre due to force of gravity which acts as a restoring force

MOTION OF A SIMPLE PENDULUM

A simple pendulum also exhibits SHM. It consists of a small bob of mass '*m*' suspended from a light string of length 'ℓ' fixed at its upper end. In the equilibrium position O, the net force on the bob is zero and the bob is stationary. Now if we bring the bob to extreme position A, the net force is not zero (Fig.10.3). There is no force acting along the string as the tension in the string cancels the component of the weight $mg\cos\theta$. Hence there is no motion along this direction.

The component of the weight $mg\sin\theta$ is directed towards the mean position and acts as a restoring force. Due to this force the bob starts moving towards the mean position O. At O, the bob has got the maximum velocity and due to inertia, it does not stop at O rather it continues to move towards the extreme position B. During its motion towards point B, the velocity of the bob decreases due to restoring force. The velocity of the bob becomes zero as it reaches the point B.

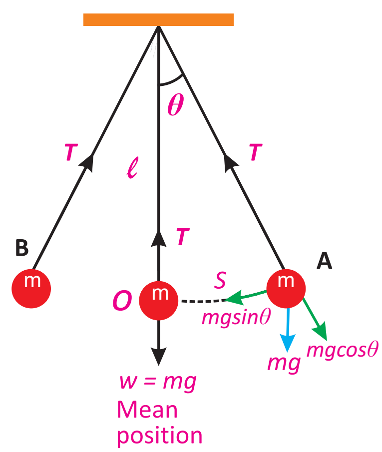


Fig. 10.3: Forces acting on a displaced pendulum. The restoring force that causes the pendulum to undergo simple harmonic motion is the component of gravitational force *mg* sinθ tangent to the path of motion

The restoring force $mg\sin\theta$ still acts towards the mean position O and due to this force the bob again starts moving towards the mean position O. In this way, the bob continues its to and fro motion about the mean position O.

It is clear from the above discussion that the speed of the bob increases while moving from point A to O due to the restoring force which acts towards O. Therefore, acceleration of the bob is also directed towards O. Similarly, when the bob moves from O to B, its speed decreases due to restoring force which again acts towards O. Therefore, acceleration of the bob is again directed towards O. It follows that the acceleration of the bob is always directed towards the mean position O. Hence the motion of a simple pendulum is SHM.

We have the following formula for the time period of a simple pendulum

$$T = 2\pi\sqrt{\frac{\ell}{g}} \quad \text{......... (10.4)}$$

From the motion of these simple systems, we can define SHM as:

Simple harmonic motion occurs when the net force is directly proportional to the displacement from the mean position and is always directed towards the mean position.

In other words, when an object oscillates about a fixed position (mean position) such that its acceleration is directly proportional to its displacement from the mean position and is always directed towards the mean position, its motion is called SHM.

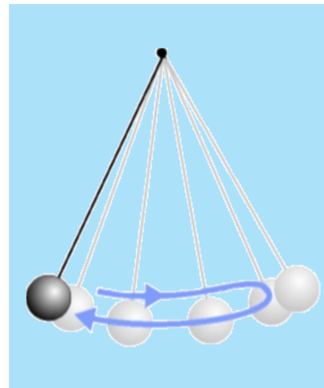
Important features of SHM are summarized as:

i. A body executing SHM always vibrates about a fixed position.

ii. Its acceleration is always directed towards the mean position.

iii. The magnitude of acceleration is always directly proportional to its displacement from the mean

Time Period



Time period of a pendulum is the time to complete one cycle.

For your information

The period of a pendulum is independent of its mass and amplitude.

Check Your Understanding

Tell whether or not these motions are examples of simple harmonic motion:
(a) up and down motion of a leaf in water pond (b) motion of a ceiling fan (c) motion of hands of clock (d) motion of a plucked string fixed at both its ends (e) movement of honey bee.

position i.e., acceleration will be zero at the mean position while it will be maximum at the extreme positions.

iv. Its velocity is maximum at the mean position and zero at the extreme positions.

Now we discuss different terms which characterize simple harmonic motion.

Vibration: One complete round trip of a vibrating body about its mean position is called one vibration.

Time Period (T): The time taken by a vibrating body to complete one vibration is called time period.

Frequency (f): The number of vibrations or cycles of a vibrating body in one second is called its frequency. It is reciprocal of time period i.e., $f = 1/T$

Amplitude (A): The maximum displacement of a vibrating body on either side from its mean position is called its amplitude.

Example 10.1: Find the time period and frequency of a simple pendulum 1.0 m long at a location where $g = 10.0\ \text{m s}^{-2}$.

Solution: Given, $\ell = 1.0\ \text{m}$, $g = 10.0\ \text{m s}^{-2}$.

Using the formula,

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

By putting the values

$$T = 2\times 3.14 \times \sqrt{\frac{1.0\ \text{m}}{10.0\ \text{m s}^{-2}}} = 1.99\ \text{s}$$

Frequency of simple pendulum is given by

$f = 1/T = 1/1.99\ \text{s} = 0.50\ \text{Hz}$

For your information

Christian Huygens invented the pendulum clock in 1656. He was inspired by the work of Galileo who had discovered that all pendulums of the same length took the same amount of time to complete one full swing. Huygens developed the first clock that could accurately measure time.

10.2 DAMPED OSCILLATIONS

Vibratory motion of ideal systems in the absence of any friction or resistance continues indefinitely under the action of a restoring force. Practically, in all systems, the force of friction retards the motion, so the systems do not oscillate indefinitely. The friction reduces the mechanical energy of the system as time passes, and the motion is said to be **damped**. This damping progressively reduces the amplitude of the vibration of motion as shown in Fig. 10.4.

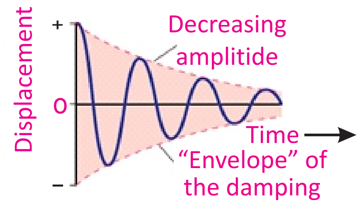


Fig. 10.4: The variation of amplitude with time of damping system

Shock absorbers in automobiles are one practical application of damped motion. A shock absorber consists of a piston moving through a liquid such as oil (Fig.10.5). The upper part of the shock absorber is firmly attached to the body of the car. When the car travels over a bump on the road, the car may vibrate violently. The shock absorbers damp these vibrations and convert their energy into heat energy of the oil. Thus

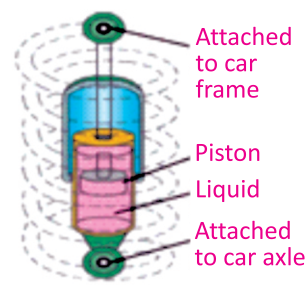


Fig. 10.5: Shock absorber

The oscillations of a system in the presence of some resistive force are damped oscillations.

10.3 WAVE MOTION

Waves play an important role in our daily life. It is because waves are carrier of energy and information over large distances. Waves require some oscillating or vibrating source. Here we demonstrate the production and propagation of different waves with the help of vibratory motion of objects.

Activity 10.1: Dip one end of a pencil into a tub of water, and move it up and down vertically (Fig. 10.6). The disturbance in the form of ripples produces water waves, which move away from the source. When the wave reaches a small piece of cork floating near the disturbance, it moves up and down about its original position while the wave will travel outwards. The net displacement of the cork is zero. The cork repeats its vibratory motion about its mean position.

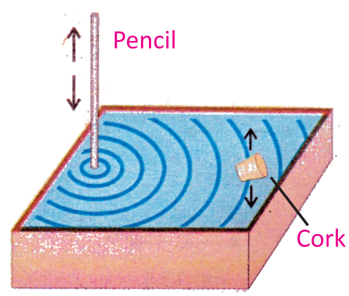


Fig. 10.6: Waves produced by dipping a pencil in a water tub

Activity 10.2: Take a rope and mark a point P on it. Tie one end of the rope with a support and stretch the rope by holding its other end in your hand (Fig. 10.7). Now, flipping the rope up and down regularly will set up a wave in the rope which will travel towards the fixed end. The point P on the rope will start vibrating up and down as the wave passes across it. The motion of point P will be perpendicular to the direction of the motion of wave.

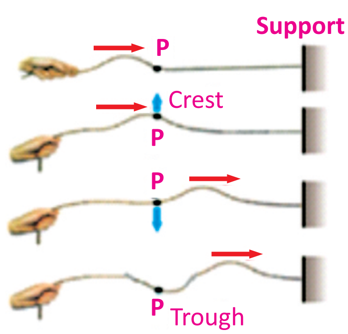


Fig. 10.7: Waves produced in a rope

From the above simple activities, we can define wave as:

A wave is a disturbance in the medium which causes the particles of the medium to undergo vibratory motion about their mean position in equal intervals of time.

There are two categories of waves:

1. Mechanical waves
2. Electromagnetic waves

Mechanical Waves: Waves which require any medium for their propagation are called mechanical waves.

Examples of mechanical waves are water waves, sound waves and waves produced on the strings and springs.

Electromagnetic Waves: Waves which do not require any medium for their propagation are called electromagnetic waves.

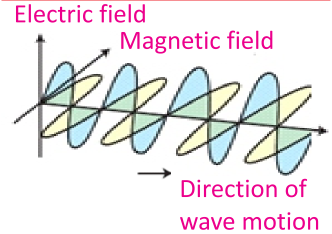
Radiowaves, television waves, X-rays, heat and light waves are some examples of electromagnetic waves.

10.4 TYPES OF MECHANICAL WAVES

Depending upon the direction of displacement of medium with respect to the direction of the propagation of wave itself, mechanical waves may be classified as longitudinal or transverse.

Longitudinal waves can be produced on a spring (slinky) placed on a smooth floor or a long bench. Fix one end of the slinky with a rigid support and hold the other end into your hand. Now give it a regular push and pull quickly in the direction of its length (Fig.10.8).

For your information



Electromagnetic waves consist of electric and magnetic fields oscillating perpendicular to each other.

Quick Quiz

Do mechanical waves pass through vacuum, that is, empty space?

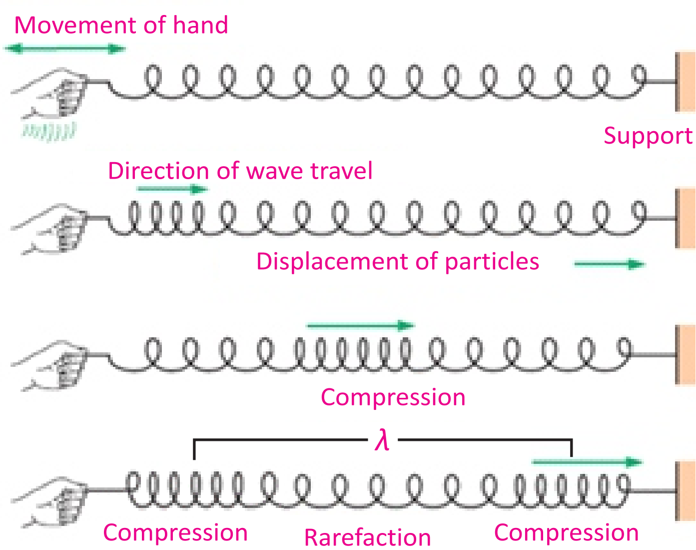


Fig. 10.8: Longitudinal wave on a slinky

A series of disturbances in the form of waves will start moving along the length of the slinky. Such a wave consists of regions called **compressions**, where the loops of the spring are close together, alternating with regions called **rarefactions** (expansions), where the loops are spaced apart. In the regions of compression, particles of the medium are closer together while in the regions of rarefaction, particles of the medium are spaced apart. The distance between two consecutive compressions is called wavelength. The compressions and rarefactions move back and forth along the direction of motion of the wave. Such a wave is called longitudinal wave and is defined as:

In longitudinal waves the particles of the medium move back and forth along the direction of propagation of wave.

We can produce transverse waves with the help of a slinky. Stretch out a slinky along a smooth floor with one end fixed. Grasp the other end of the slinky and move it up and down quickly (Fig.10.9). A wave in the form of alternate crests and troughs will start travelling towards the fixed end. The crests are the highest points while the troughs are the lowest points of the particles of the medium from the mean position. The distance between two consecutive crests or troughs is called

For your Information

Longitudinal waves move faster through solids than through gases or liquids. Transverse waves move through solids at a speed of less than half of the speed of longitudinal waves. It is because the restoring force exerted during this up and down motion of particles of the medium is less than the restoring force exerted by a back and forth motion of particles of the medium in case of longitudinal waves.

wavelength. The crests and troughs move perpendicular to the direction of the wave.

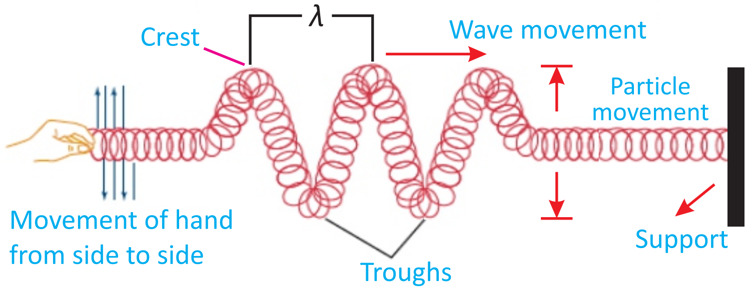


Fig. 10.9: Transverse wave on a slinky

Therefore, transverse waves can be defined as:

In case of transverse waves, the vibratory motion of particles of the medium is perpendicular to the direction of propagation of waves.

Waves on the surface of water and light waves are examples of transverse waves.

WAVES AS CARRIERS OF ENERGY

Energy can be transferred from one place to another through waves. For example, when we shake the stretched string up and down, we provide our muscular energy to the string. As a result, a set of waves can be seen travelling along the string. The vibrating force from the hand disturbs the particles of the string and sets them in motion. These particles then transfer their energy to the adjacent particles in the string. Energy is thus transferred from one place of the medium to the other in the form of wave.

The amount of energy carried by the wave depends on the distance of the stretched string from its rest position. That is, the energy in a wave depends on the amplitude of the wave. If we shake the string faster, we give more energy per second to produce wave of higher frequency, and the wave delivers more energy per second to the particles of the string as it moves forward.

Water waves also transfer energy from one place to another as explained below:

Activity 10.3: Drop a stone into a pond of water. Water waves will be produced on the surface of water and will travel outwards (Fig. 10.10). Place a cork at some distance from the falling stone. When waves reach the cork, it will move up and down alongwith the motion of the water particles by getting energy from the waves.

Fig. 10.10

This activity shows that water waves like other waves transfer energy from one place to another without transferring matter, i.e., water.

For your information

Generating a high frequency wave, requires more energy per second than to generate a low frequency wave. Thus, a high frequency wave carries more energy than a low frequency wave of the same amplitude.

RELATION BETWEEN VELOCITY, FREQUENCY AND WAVELENGTH

Wave is a disturbance in a medium which travels from one place to another and hence has a specific velocity of travelling. This is called the velocity of wave which is defined by

Velocity = distance/time

$$v = \frac{d}{t}$$

If time taken by the wave in moving from one point to another is equal to its time period T, then the distance covered by the wave will be equal to one wavelength λ, hence we can write:

$$v = \frac{\lambda}{T}$$

But time period T, is reciprocal of the frequency f, i.e., $T = \frac{1}{f}$

Do you know?

Earthquake produces waves through the crust of the Earth in the form of seismic waves. By studying such waves, the geophysicists learn about the internal structure of the Earth and information about the occurrence of future Earth activity.

Therefore, $v = f\lambda$ .... (10.5)

Eq. (10.5) is true both for longitudinal and transverse waves.

Example 10.2: A wave moves on a slinky with frequency of 4 Hz and wavelength of 0.4 m. What is the speed of the wave?

Solution: Given that, $f = 4$ Hz, $\lambda = 0.4$ m

Wave speed $v = f\lambda$

$= (4\text{ Hz})(0.4\text{ m})$

$v = 1.6\text{ m s}^{-1}$

10.5 RIPPLE TANK

Ripple tank is a device to produce water waves and to study their characteristics.

This apparatus consists of a rectangular tray having glass bottom and is placed nearly half metre above the surface of a table (Fig. 10.11). Waves can be produced on the surface of water present in the tray by means of a vibrator (paddle).

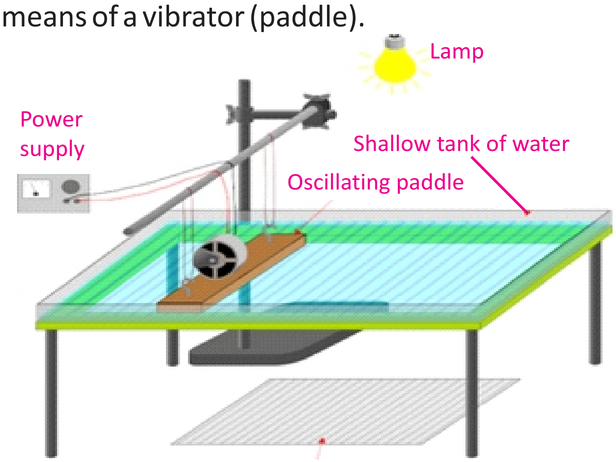


Fig. 10.11: Ripple tank apparatus

This vibrator is an oscillating electric motor fixed on a wooden plate over the tray such that its lower surface just touches the surface of water. On setting the vibrator ON, this wooden plate starts vibrating to generate water waves consisting of straight wavefronts (Fig.10.12). An electric bulb is hung above the tray to observe the image of water waves on the paper or screen. The crests and troughs of the waves appear as bright and dark lines respectively, on the screen.

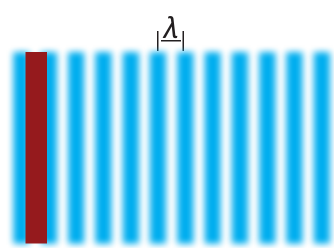


Fig. 10.12: Waves consisting of straight wavefronts

Now we explain the reflection of water waves with the help of ripple tank.

Place a barrier in the ripple tank. The water waves will reflect from the barrier. If the barrier is placed at an angle to the wavefront, the reflected waves can be seen to obey the law of reflection i.e., the angle of the incident wave along the normal will be equal to the angle of the reflected wave (Fig.10.13). Thus, we define reflection of waves as:

When waves moving in one medium fall on the surface of another medium they bounce back into the first medium such that the angle of incidence is equal to the angle of reflection.

The speed of a wave in water depends on the depth of water. If a block is submerged in the ripple tank, the depth of water in the tank will be shallower over the block than elsewhere. When water waves enter the region of shallow water their wavelength decreases (Fig.10.14). But the frequency of the water waves remains the same in both parts of water because it is equal to the frequency of the vibrator.

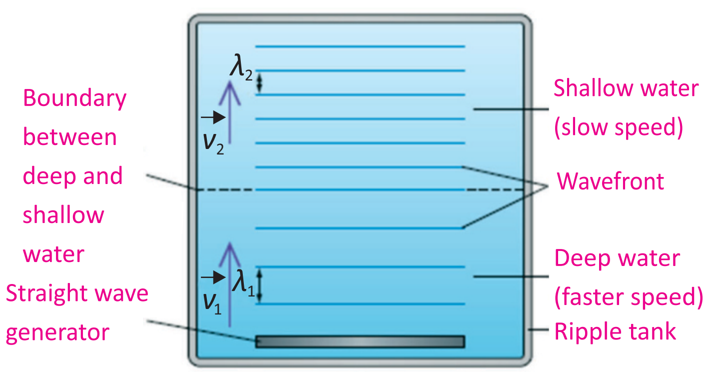


Fig. 10.14

For the observation of refraction of water waves, we repeat the above experiment such that the boundary between the deep and the shallower water is at some angle to the wavefront (Fig. 10.15). Now we will observe that in addition to the change in wavelength, the waves change their direction of propagation as well. Note that the direction of propagation is always normal to the wavefronts. This change of path of water waves while passing from a region of deep water to that of shallower one is called refraction which is defined as:

Quick Quiz
What do the dark and bright fringes on the screen of ripple tank represent?

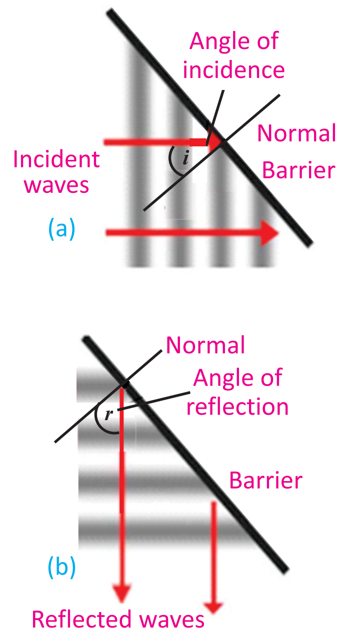


Fig. 10.13: Reflection of water waves from a plane barrier

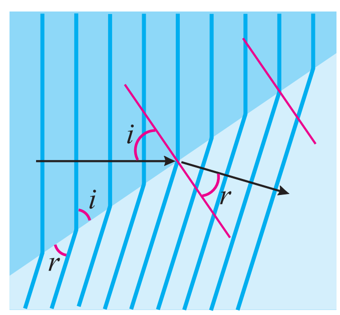


Fig. 10.15: Refraction of water waves

When a wave from one medium enters into the second medium at some angle, its direction of travel changes.

Now we observe the phenomenon of diffraction of water waves. Generate straight waves in a ripple tank and place two obstacles in line in such a way that separation between them is equal to the wavelength of water waves. After passing through a small slit between the two obstacles, the waves will spread in every direction and change into almost semicircular pattern (Fig. 10.16). Diffraction of waves can only be observed clearly if the size of the obstacle is comparable with the wavelength of the wave. Fig.10.17 shows the diffraction of waves while passing through a slit with size larger than the wavelength of the wave. Only a small diffraction occurs near the corners of the obstacle.

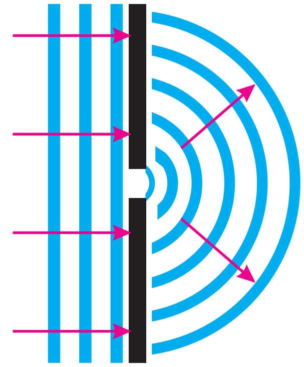


Fig.10.16: Diffraction of water waves through a small slit

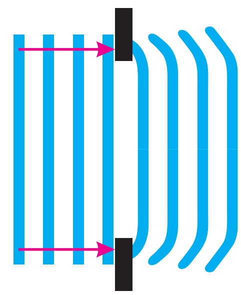


Fig.10.17: Diffraction of water waves through a large slit

The bending or spreading of waves around the sharp edges or corners of obstacles or slits is called diffraction.

Example 10.3: A student performs an experiment with waves in water. The student measures the wavelength of a wave to be 10 cm. By using a stopwatch and observing the oscillations of a floating ball, the student measures a frequency of 2 Hz. If the student starts a wave in one part of a tank of water, how long will it take the wave to reach the opposite side of the tank 2 m away?

Solution:

(1) We are given the frequency, wavelength, and distance.

(2) We have to calculate the time, the wave takes to move a distance of 2 m.

(3) The relationship between frequency, wavelength, and speed is $v = f\lambda$. The relationship between time, speed, and distance is $v = d/t$

(4) Rearrange the speed formula to solve for the time: $t = d/v$

The speed of the wave is the frequency times the wavelength.

$$v = f\lambda = (2\ \text{Hz})(0.1\ \text{m}) = 0.2\ \text{m s}^{-1}.$$

Use this value to calculate the time:

$$t = 2\ \text{m}/0.2\ \text{m s}^{-1} = 10\ \text{s}$$

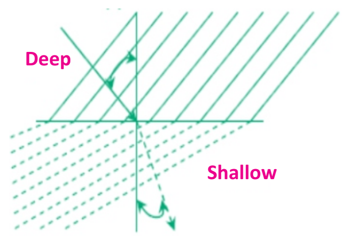


Fig.10.18

ACTIVITY

Study **Fig. 10.18** to answer the following questions:

1. What happens to the direction of wave when water waves pass from deep to shallow part of the water?

2. Are the magnitudes of angle of incidence and angle of refraction equal?

3. Which will be greater?