NETWORK THEOREMS

9

9.1 INTRODUCTION

This chapter introduces a number of theorems that have application throughout the field of electricity and electronics. Not only can they be used to solve networks such as encountered in the previous chapter, but they also provide an opportunity to determine the impact of a particular source or element on the response of the entire system. In most cases, the network to be analyzed and the mathematics required to find the solution are simplified. All of the theorems appear again in the analysis of ac networks. In fact, the application of each theorem to ac networks is very similar in content to that found in this chapter.

The first theorem to be introduced is the superposition theorem, followed by Thévenin's theorem, Norton's theorem, and the maximum power transfer theorem. The chapter concludes with a brief introduction to Millman's theorem and the substitution and reciprocity theorems.

9.2 SUPERPOSITION THEOREM

The **superposition theorem** is unquestionably one of the most powerful in this field.

In general, the theorem can be used to do the following:

- ***Analyze networks such as introduced in the last chapter that have two or more sources that are not in series or parallel.***
- ***Reveal the effect of each source on a particular quantity of interest.***
- ***For sources of different types (such as dc and ac which affect the parameters of the network in a different manner), apply a separate analysis for each type, with the total result simply the algebraic sum of the results.***

The superposition theorem states the following:

The current through, or voltage across, any element of a network is equal to the algebraic sum of the currents or voltages produced independently by each source.

In other words, this theorem allows us to find a solution for a current or voltage using *only one source at a time.* Once we have the solution for each source, we can combine the results to obtain the total solution. The term *algebraic* appears in the above theorem statement because the currents resulting from the sources of the network can have different directions, just as the resulting voltages can have opposite polarities.

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately. However, to apply the superposition principle, we must keep two things in mind:

1. We consider one independent source at a time while all other independent sources are *turned off*. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit). This way we obtain a simpler and more manageable circuit.
2. Dependent sources are left intact because they are controlled by circuit variables.

With these in mind, we apply the superposition principle in three steps:

If we are to consider the effects of each source, the other sources obviously must be removed. Setting a voltage source to zero volts is like placing a short circuit across its terminals. Therefore,

when removing a voltage source from a network schematic, replace it with a direct connection (short circuit) of zero ohms. Any internal resistance associated with the source must remain in the network.

Setting a current source to zero amperes is like replacing it with an open circuit. Therefore,

when removing a current source from a network schematic, replace it by an open circuit of infinite ohms. Any internal resistance associated with the source must remain in the network.

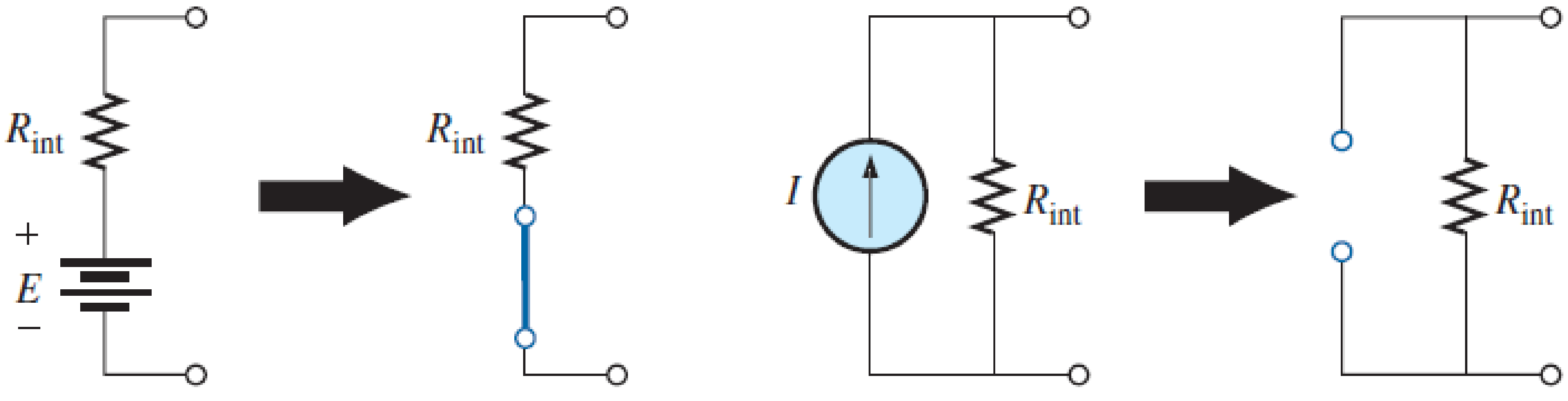


FIG. 9.1

Removing a voltage source and a current source to permit the application of the superposition theorem.

Since the effect of each source will be determined independently, the number of networks to be analyzed will equal the number of sources.

If a particular current of a network is to be determined, the contribution to that current must be determined for *each source*. When the effect of each source has been determined, those currents in the same direction are added, and those having the opposite direction are subtracted; the algebraic sum is being determined. The total result is the direction of the larger sum and the magnitude of the difference.

Similarly, if a particular voltage of a network is to be determined, the contribution to that voltage must be determined for each source. When the effect of each source has been determined, those voltages with the same polarity are added, and those with the opposite polarity are subtracted; the algebraic sum is being determined. The total result has the polarity of the larger sum and the magnitude of the difference.

Superposition cannot be applied to power effects because the power is related to the square of the voltage across a resistor or the current through a resistor. The squared term results in a nonlinear (a curve, not a straight line) relationship between the power and the determining current or voltage. For example, doubling the current through a resistor does not double the power to the resistor (as defined by a linear relationship) but, in fact, increases it by a factor of 4 (due to the squared term). Tripling the current increases the power level by a factor of 9. Example 9.3 demonstrates the differences between a linear and a nonlinear relationship.

EXAMPLE 9.1 Using the superposition theorem, determine current I_1 for the network in Fig. 9.2.

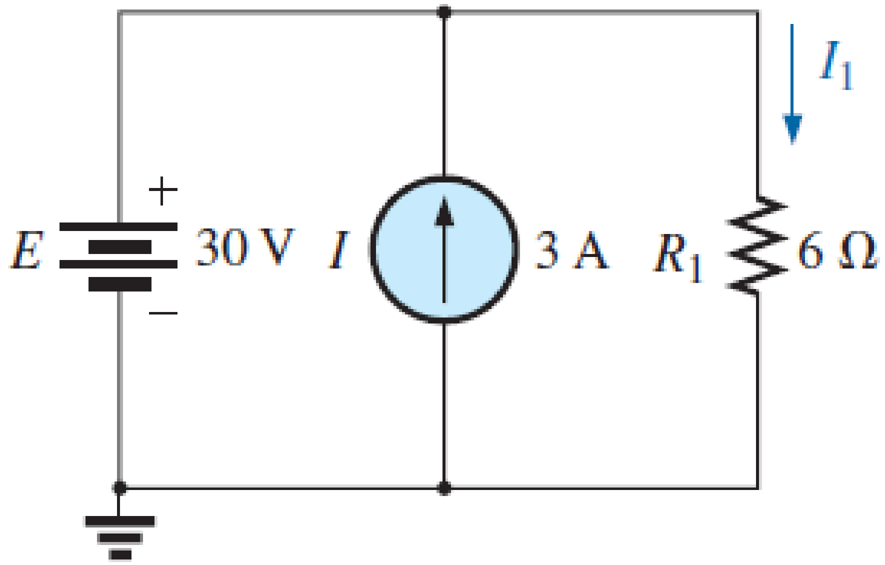
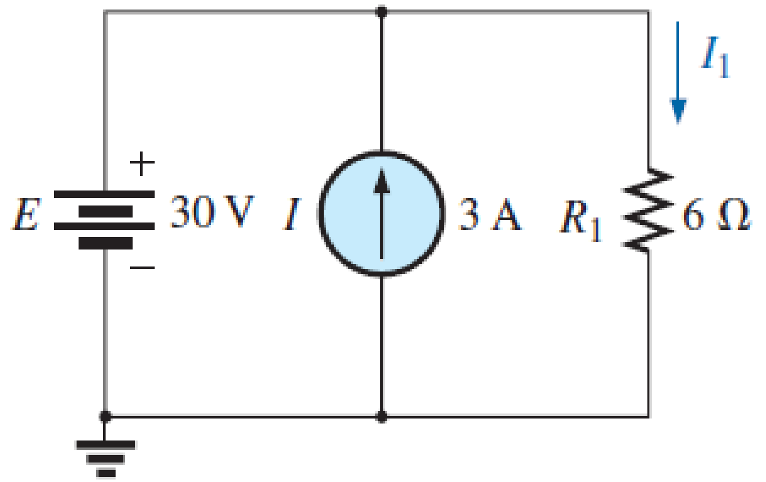
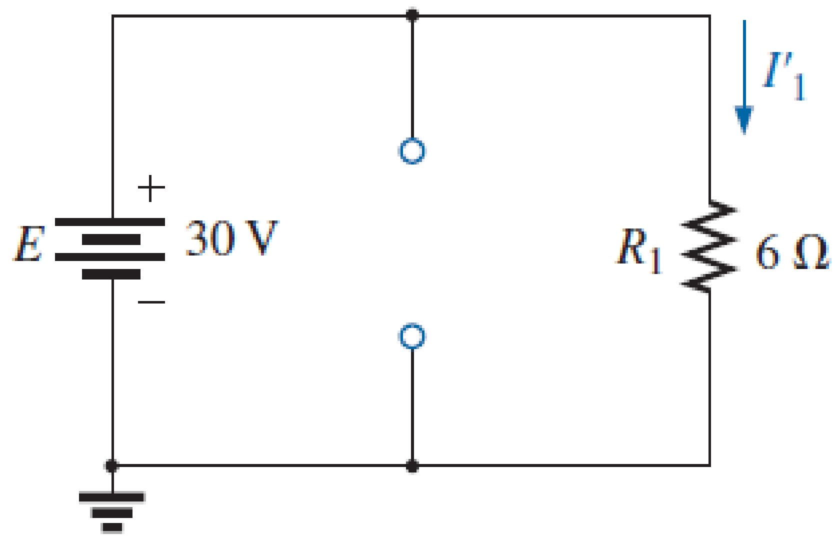


FIG. 9.2
Two-source network to be analyzed using the superposition theorem in Example 9.1.



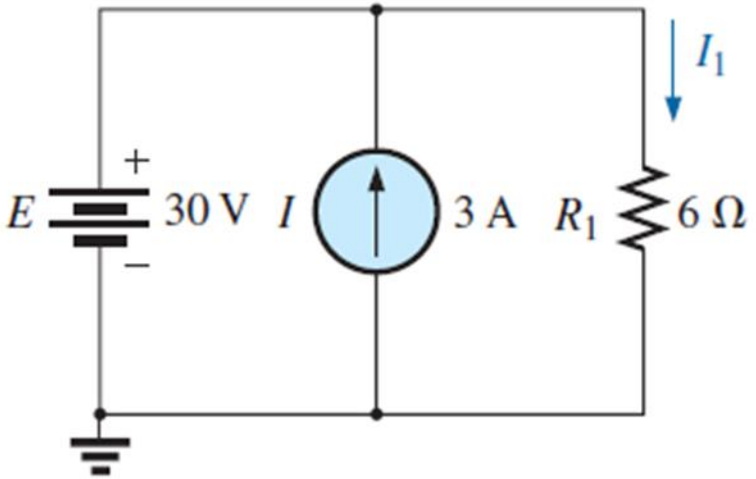


Determining the effect of the 30 V supply on the current I_1 in Fig. 9.2.

Solution: Since two sources are present, there are two networks to be analyzed. First let us determine the effects of the voltage source by setting the current source to zero amperes as shown in Fig. 9.3. Note that the resulting current is defined as I'_1 because it is the current through resistor R_1 due to the voltage source only.

Due to the open circuit, resistor R_1 is in series (and, in fact, in parallel) with the voltage source E. The voltage across the resistor is the applied voltage, and current I'_1 is determined by

$$I'_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{30\text{ V}}{6\ \Omega} = 5\text{ A}$$



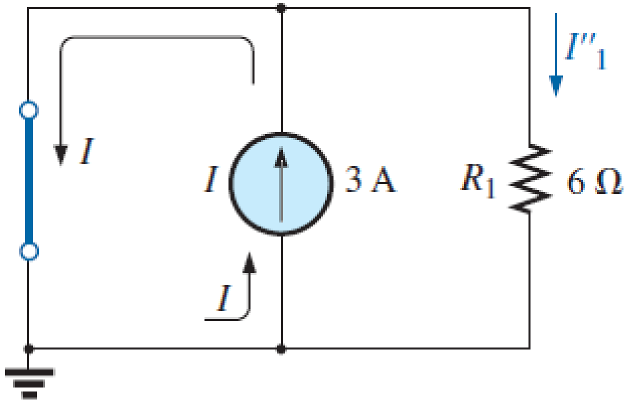
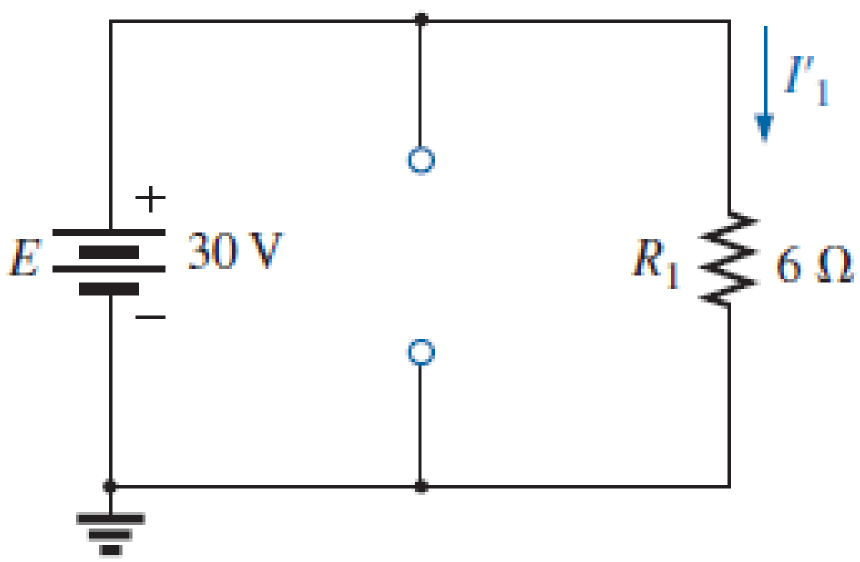
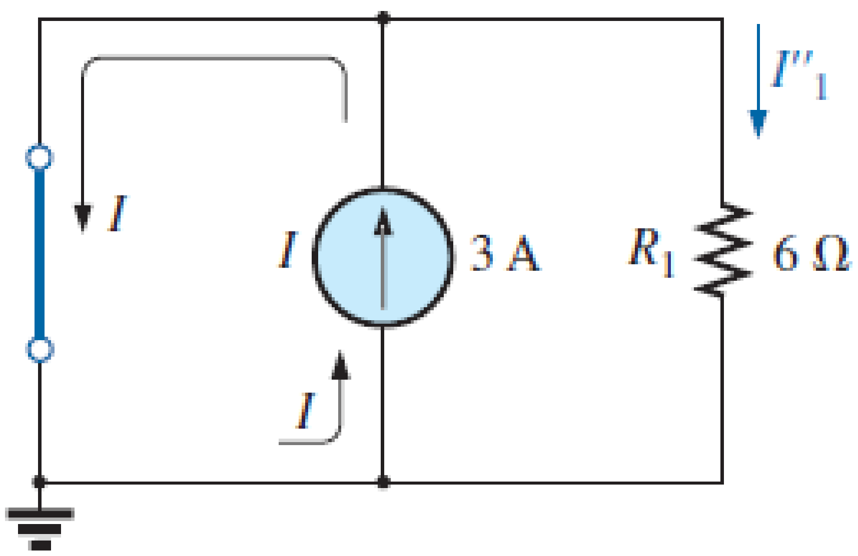


FIG. 9.4
Determining the effect of the 3 A current source on the current I_1 in Fig. 9.2.

Now for the contribution due to the current source. Setting the voltage source to zero volts results in the network in Fig. 9.4, which presents us with an interesting situation. The current source has been replaced with a short-circuit equivalent that is directly across the current source and resistor R_1. Since the source current takes the path of least resistance, it chooses the zero ohm path of the inserted short-circuit equivalent, and the current through R_1 is zero amperes. This is clearly demonstrated by an application of the current divider rule as follows:

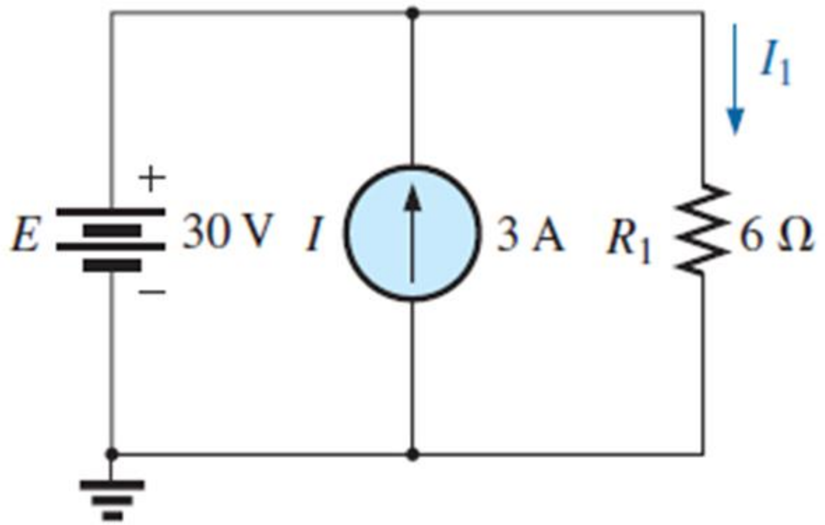
$$I''_1 = \frac{R_{sc}I}{R_{sc} + R_1} = \frac{(0\ \Omega)I}{0\ \Omega + 6\ \Omega} = 0\ \text{A}$$





Since I'_1 and I''_1 have the same defined direction in Figs. 9.3 and 9.4, the total current is defined by

$$I_1 = I'_1 + I''_1 = 5\text{ A} + 0\text{ A} = \mathbf{5\ A}$$



Although this has been an excellent introduction to the application of the superposition theorem, it should be immediately clear in Fig. 9.2 that the voltage source is in parallel with the current source and load resistor R_1, so the voltage across each must be 30 V. The result is that I_1 must be determined solely by

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{30\ \text{V}}{6\ \Omega} = \mathbf{5\ A}$$

EXAMPLE 9.2 Using the superposition theorem, determine the current through the 12 Ω resistor in Fig. 9.5.

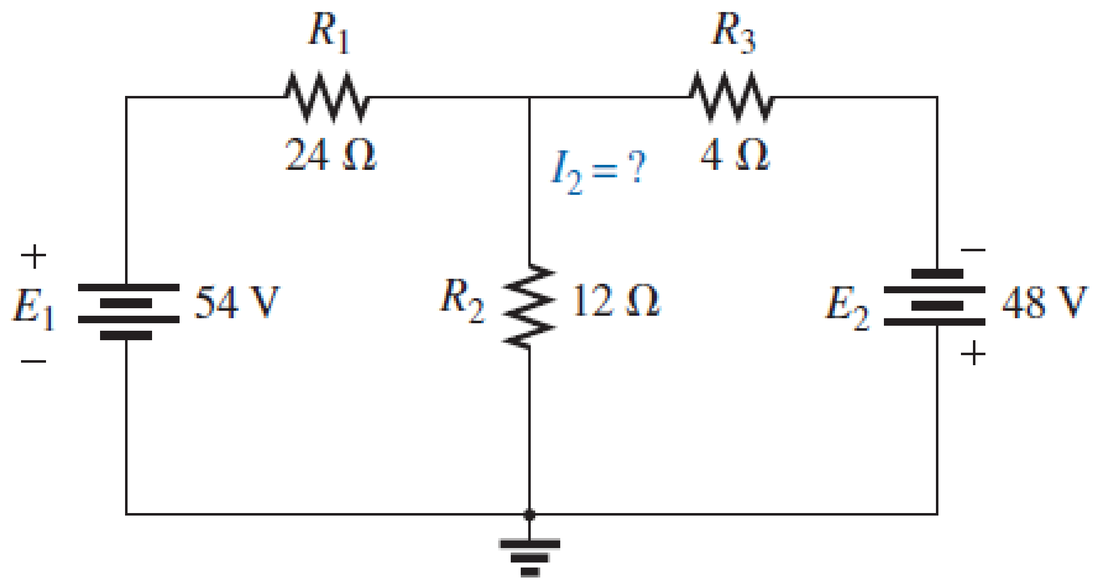


FIG. 9.5

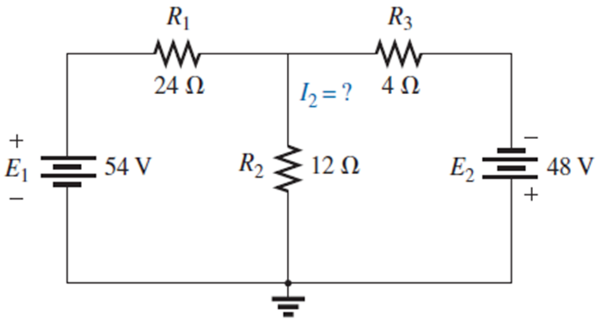


FIG. 9.5

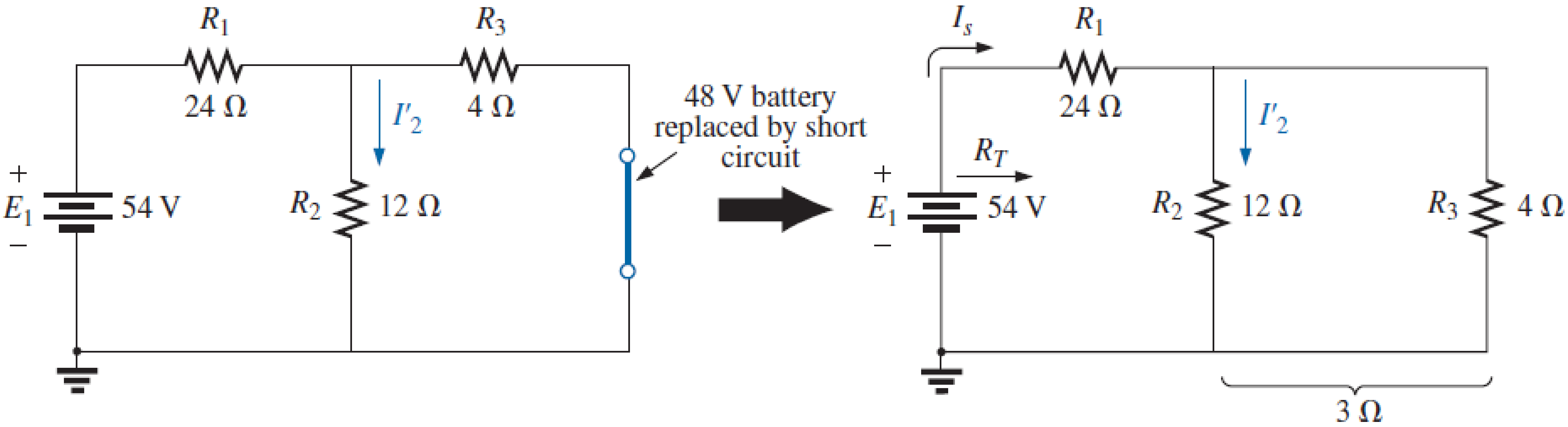


FIG. 9.6

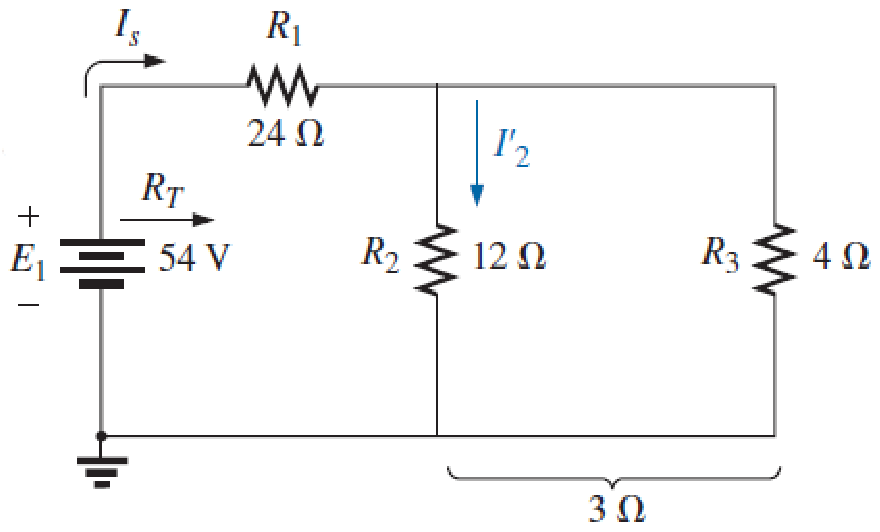
Using the superposition theorem to determine the effect of the 54 V voltage source on current I_2 in Fig. 9.5.

The total resistance seen by the source is therefore

$$R_T = R_1 + R_2 \| R_3 = 24\ \Omega + 12\ \Omega \| 4\ \Omega = 24\ \Omega + 3\ \Omega = 27\ \Omega$$

and the source current is

$$I_s = \frac{E_1}{R_T} = \frac{54\ \text{V}}{27\ \Omega} = 2\ \text{A}$$



Using the current divider rule results in the contribution to I_2 due to the 54 V source:

$$I'_2 = \frac{R_3 I_s}{R_3 + R_2} = \frac{(4\ \Omega)(2\ \text{A})}{4\ \Omega + 12\ \Omega} = 0.5\ \text{A}$$

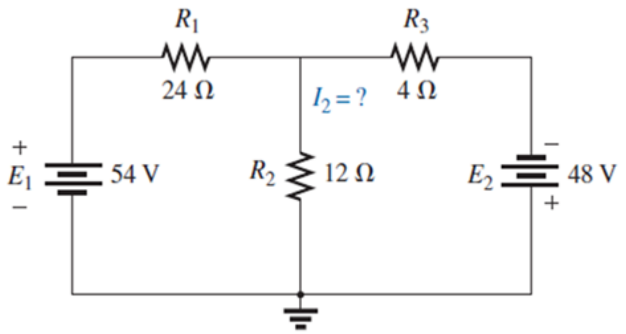


FIG. 9.5

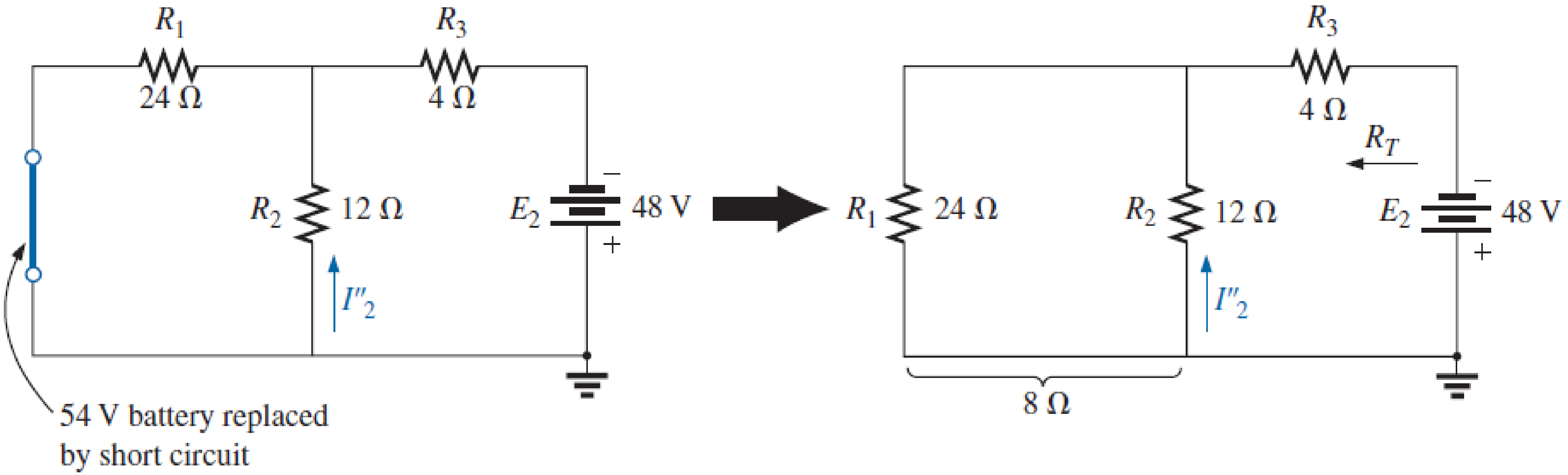


FIG. 9.7

Using the superposition theorem to determine the effect of the 48 V voltage source on current I_2 in Fig. 9.5.

Therefore, the total resistance seen by the 48 V source is

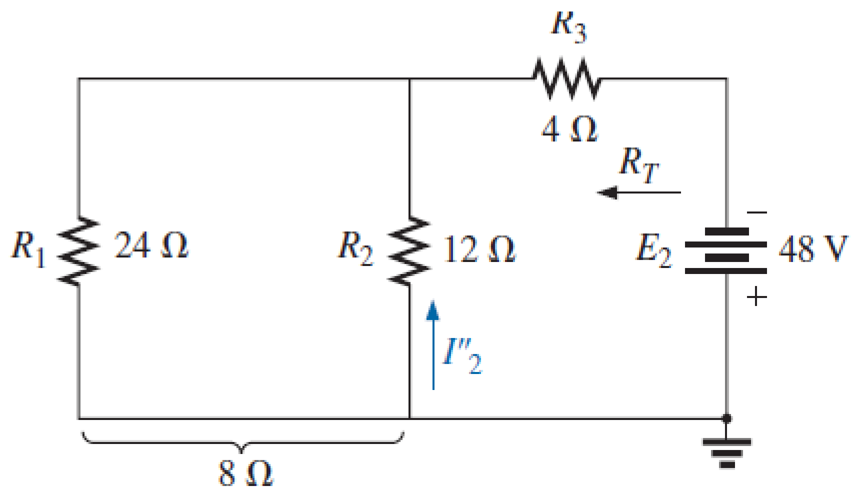
$$R_T = R_3 + R_2 \| R_1 = 4\ \Omega + 12\ \Omega \| 24\ \Omega = 4\ \Omega + 8\ \Omega = 12\ \Omega$$

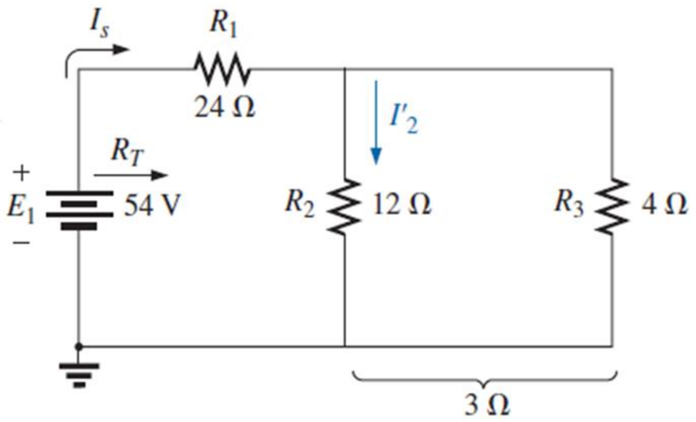
and the source current is

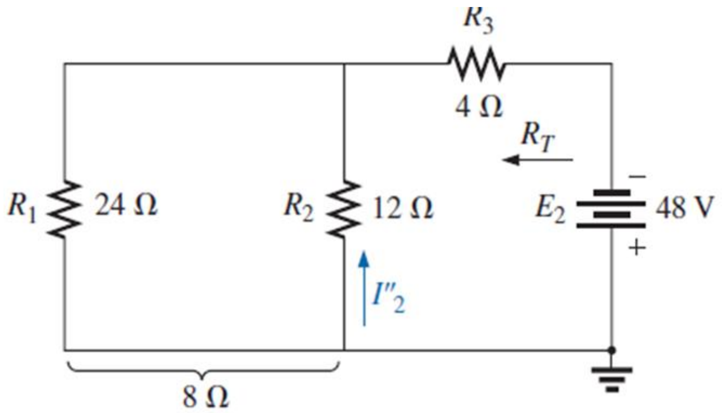
$$I_s = \frac{E_2}{R_T} = \frac{48\ \text{V}}{12\ \Omega} = 4\ \text{A}$$

Applying the current divider rule results in

$$I''_2 = \frac{R_1(I_s)}{R_1 + R_2} = \frac{(24\ \Omega)(4\ \text{A})}{24\ \Omega + 12\ \Omega} = 2.67\ \text{A}$$







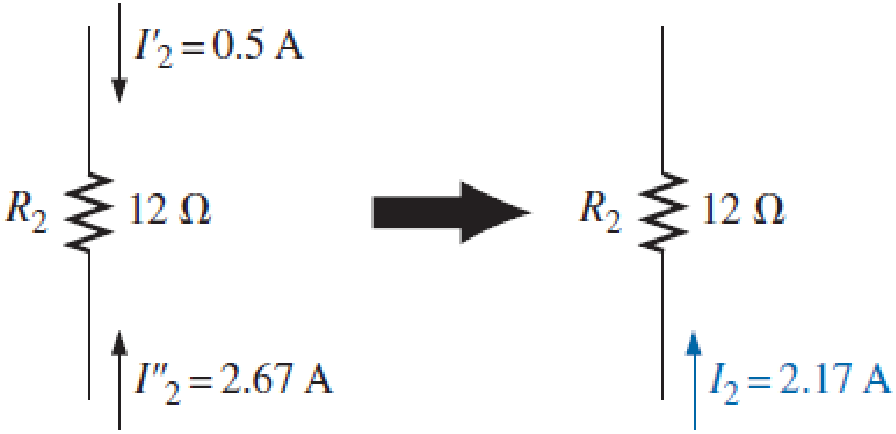


FIG. 9.8

Using the results of Figs. 9.6 and 9.7 to determine current I_2 for the network in Fig. 9.5.

It is now important to realize that current I_2 due to each source has a different direction, as shown in Fig. 9.8. The net current therefore is the difference of the two and the direction of the larger as follows:

$$I_2 = I''_2 - I'_2 = 2.67\text{ A} - 0.5\text{ A} = \mathbf{2.17\text{ A}}$$

EXAMPLE 9.3

a. Using the superposition theorem, determine the current through resistor R_2 for the network in Fig. 9.9.

b. Demonstrate that the superposition theorem is not applicable to power levels.

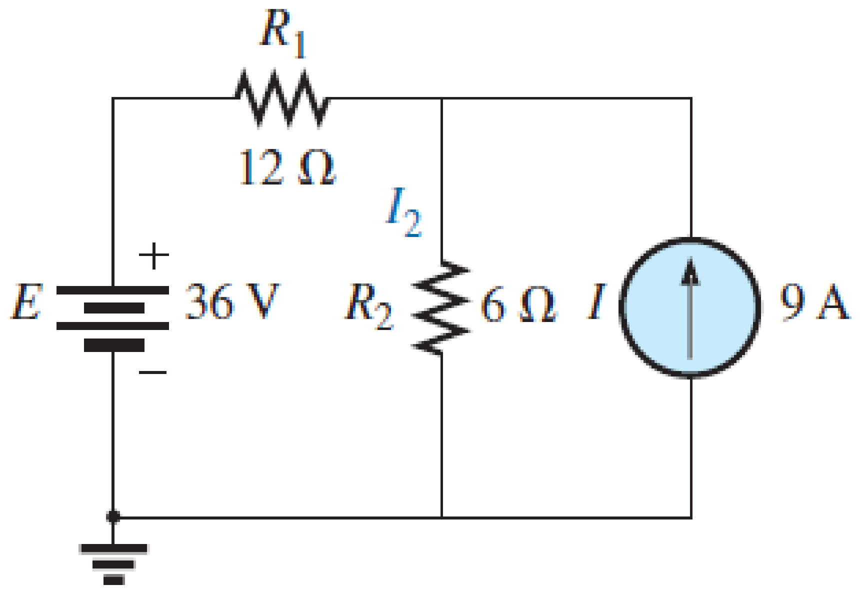


FIG. 9.9

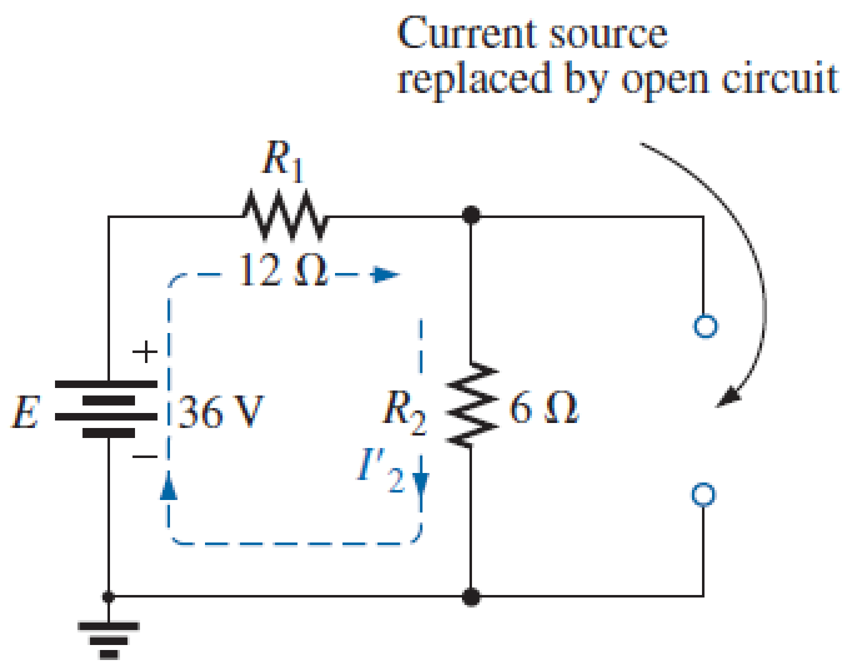


FIG. 9.10

Replacing the 9 A current source in Fig. 9.9 by an open circuit to determine the effect of the 36 V voltage source on current I_2.

$$I'_2 = \frac{E}{R_T} = \frac{E}{R_1 + R_2} = \frac{36\ \text{V}}{12\ \Omega + 6\ \Omega} = \frac{36\ \text{V}}{18\ \Omega} = 2\ \text{A}$$

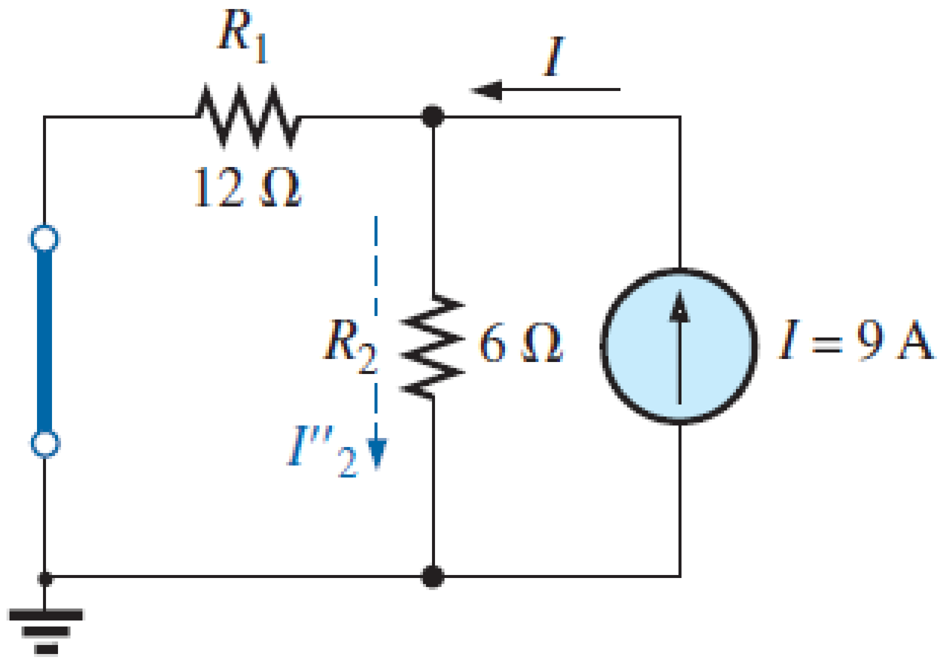


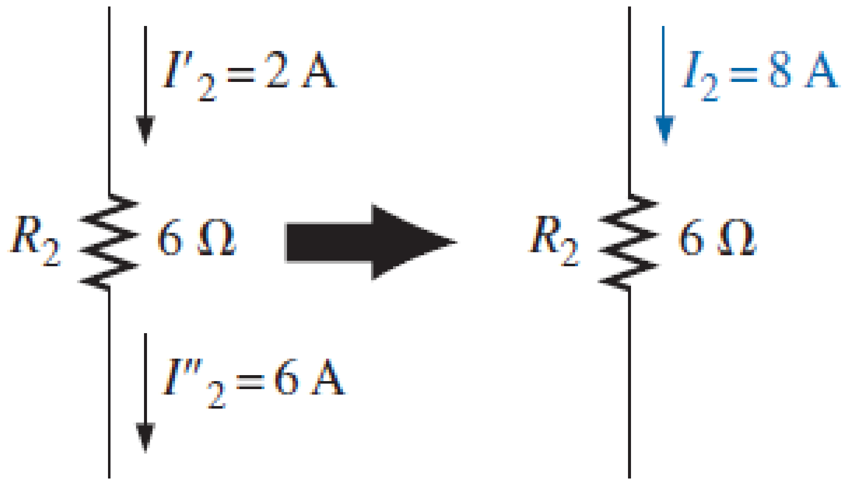
FIG. 9.11

Examining the effect of the 9 A current source requires replacing the 36 V voltage source by a short-circuit equivalent as shown in Fig. 9.11. The result is a parallel combination of resistors R_1 and R_2. Applying the current divider rule results in

$$I''_2 = \frac{R_1(I)}{R_1 + R_2} = \frac{(12\ \Omega)(9\ \text{A})}{12\ \Omega + 6\ \Omega} = 6\ \text{A}$$

Since the contribution to current I_2 has the same direction for each source, as shown in Fig. 9.12, the total solution for current I_2 is the sum of the currents established by the two sources. That is,

$$I_2 = I'_2 + I''_2 = 2\text{ A} + 6\text{ A} = \mathbf{8\ A}$$



b. Using Fig. 9.10 and the results obtained, the power delivered to the 6 Ω resistor is

$$P_1 = (I'_2)^2(R_2) = (2\text{ A})^2(6\ \Omega) = \mathbf{24\ W}$$

Using Fig. 9.11 and the results obtained, the power delivered to the 6 Ω resistor is

$$P_2 = (I''_2)^2(R_2) = (6\text{ A})^2(6\ \Omega) = \mathbf{216\ W}$$

Using the total results of Fig. 9.12, the power delivered to the 6 Ω resistor is

$$P_T = I_2^2 R_2 = (8\text{ A})^2(6\ \Omega) = \mathbf{384\ W}$$

It is now quite clear that the power delivered to the 6 Ω resistor using the total current of 8 A is not equal to the sum of the power levels due to each source independently. That is,

$$P_1 + P_2 = 24\text{ W} + 216\text{ W} = 240\text{ W} \neq P_T = 348\text{ W}$$

EXAMPLE 9.4 Using the principle of superposition, find the current I_2 through the 12 kΩ resistor in Fig. 9.15.

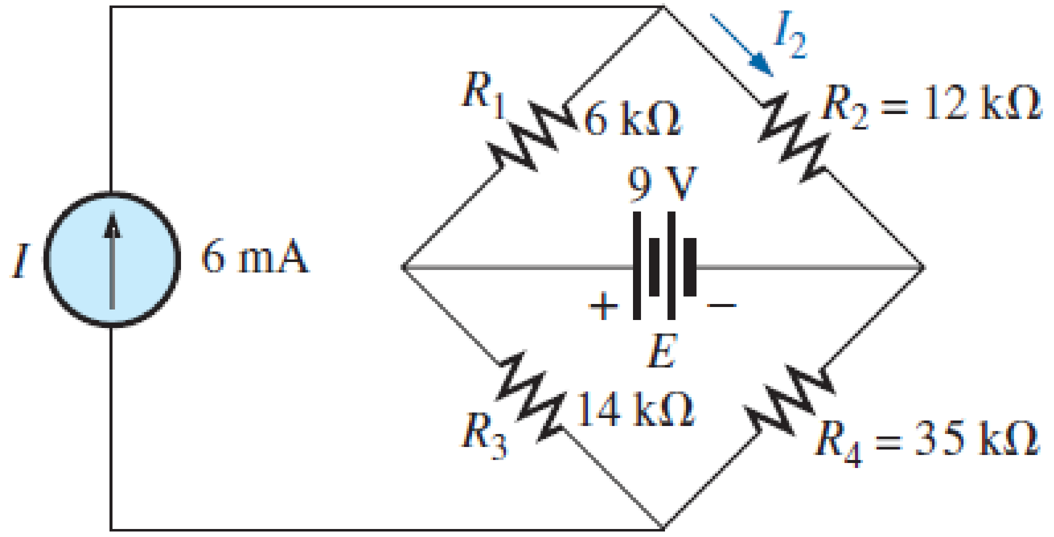


FIG. 9.15
Example 9.4.

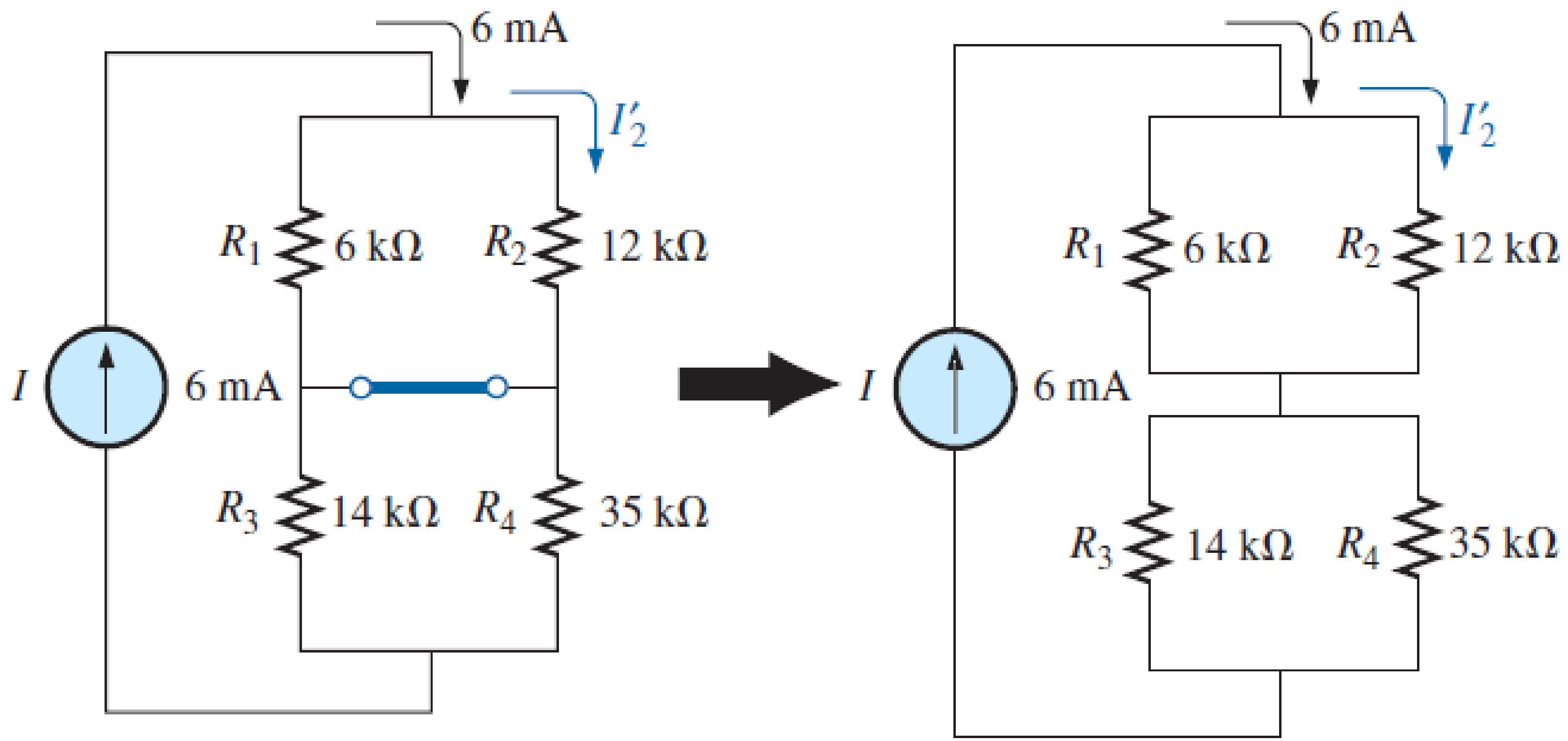


FIG. 9.16

The effect of the current source I on the current I_2.

Current divider rule:

$$I'_2 = \frac{R_1 I}{R_1 + R_2} = \frac{(6\ \text{k}\Omega)(6\ \text{mA})}{6\ \text{k}\Omega + 12\ \text{k}\Omega} = 2\ \text{mA}$$

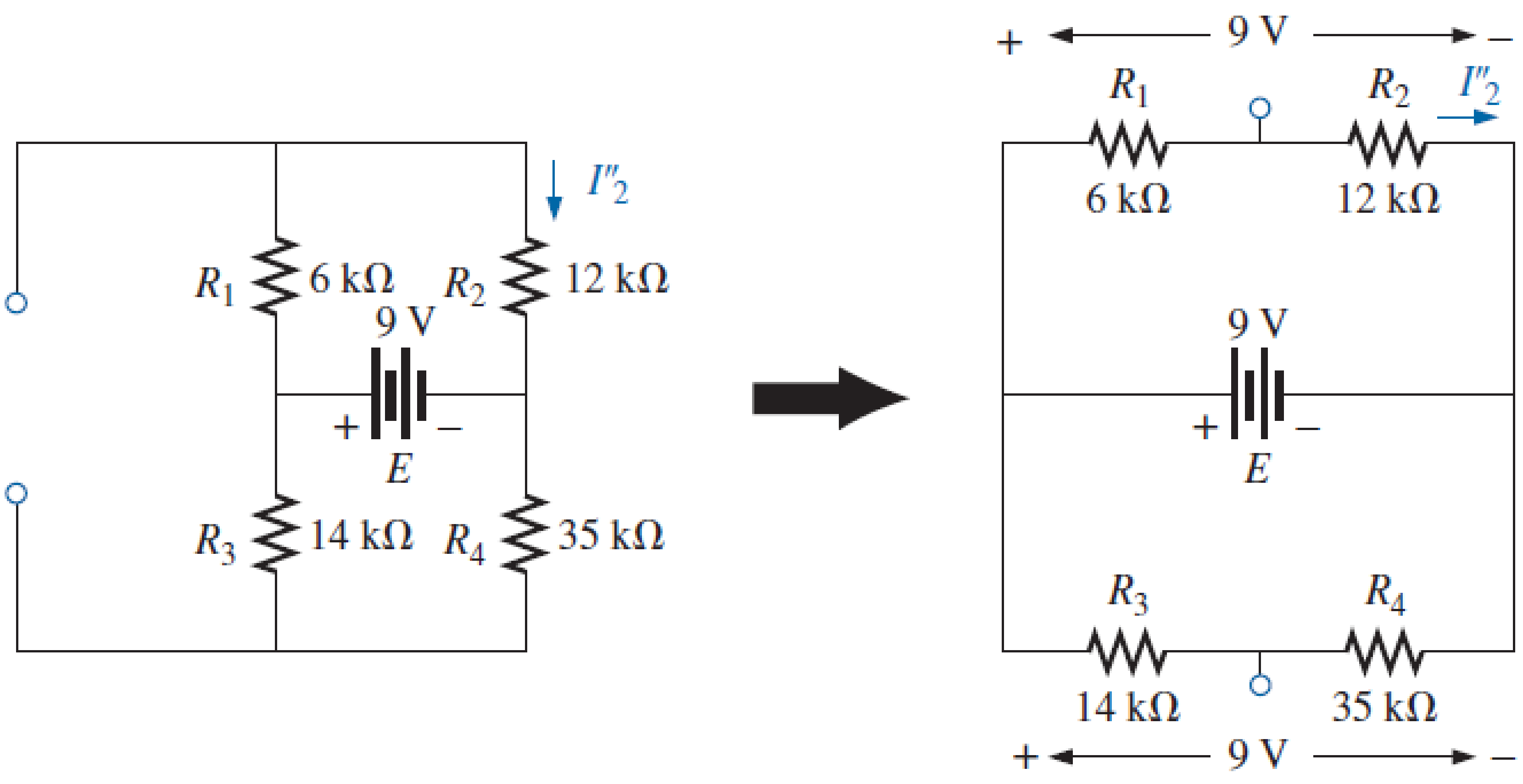


FIG. 9.17

The effect of the voltage source E on the current I_2.

Considering the effect of the 9 V voltage source (Fig 9.17):

$$I''_2 = \frac{E}{R_1 + R_2} = \frac{9\ \text{V}}{6\ \text{k}\Omega + 12\ \text{k}\Omega} = 0.5\ \text{mA}$$

Since I'_2 and I''_2 have the same direction through R_2, the desired current is the sum of the two:

$$\begin{aligned} I_2 &= I'_2 + I''_2 \\ &= 2\text{ mA} + 0.5\text{ mA} \\ &= \mathbf{2.5\text{ mA}} \end{aligned}$$

EXAMPLE 9.5 Find the current through the 2 Ω resistor of the network in Fig. 9.18. The presence of three sources results in three different networks to be analyzed.

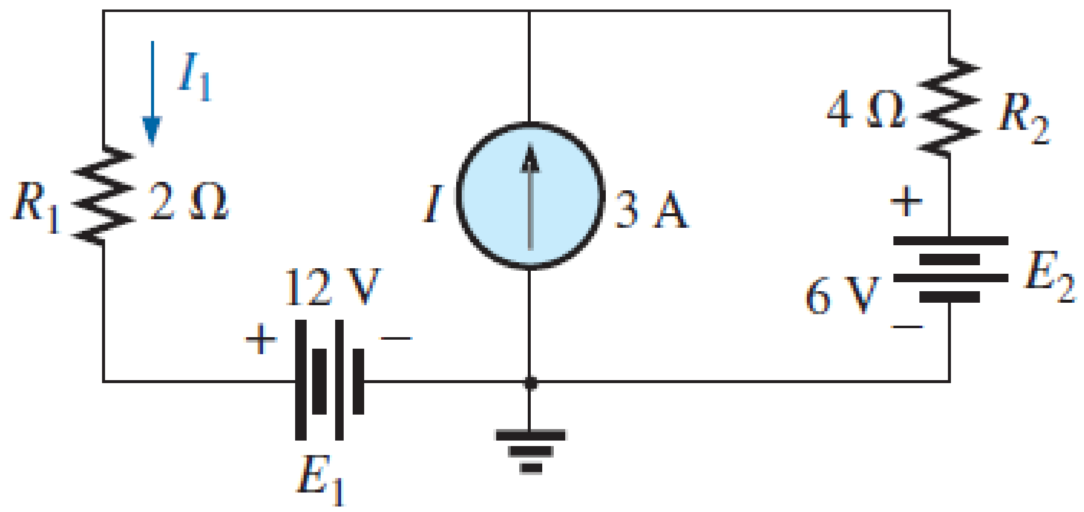


FIG. 9.18
Example 9.5.

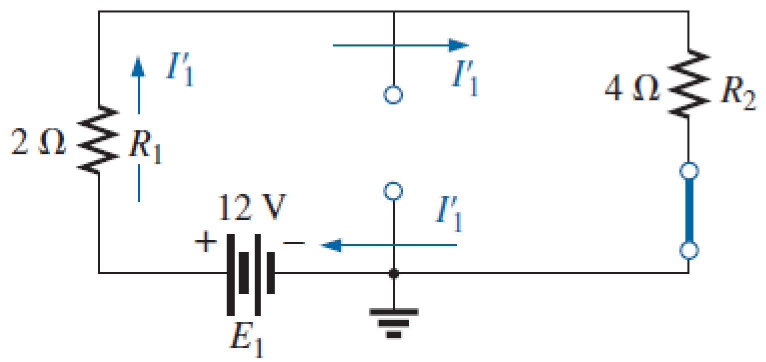


FIG. 9.19
The effect of E_1 on the current I.

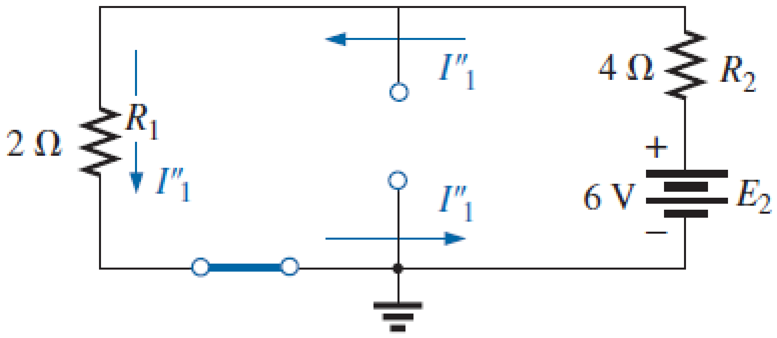


FIG. 9.20
The effect of E_2 on the current I_1.

Solution: Considering the effect of the 12 V source (Fig. 9.19):

$$I'_1 = \frac{E_1}{R_1 + R_2} = \frac{12\text{ V}}{2\ \Omega + 4\ \Omega} = \frac{12\text{ V}}{6\ \Omega} = 2\text{ A}$$

Considering the effect of the 6 V source (Fig. 9.20):

$$I''_1 = \frac{E_2}{R_1 + R_2} = \frac{6\text{ V}}{2\ \Omega + 4\ \Omega} = \frac{6\text{ V}}{6\ \Omega} = 1\text{ A}$$

Considering the effect of the 3 A source (Fig. 9.21):
Applying the current divider rule,

$$I'''_1 = \frac{R_2 I}{R_1 + R_2} = \frac{(4\ \Omega)(3\text{ A})}{2\ \Omega + 4\ \Omega} = \frac{12\text{ A}}{6} = 2\text{ A}$$

The total current through the 2 Ω resistor appears in Fig. 9.22 and

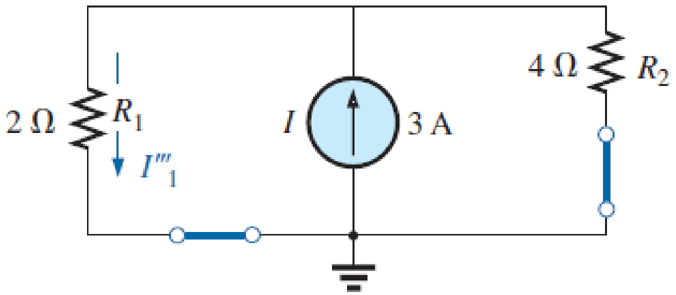


FIG. 9.21
The effect of I on the current I_1.

Same direction as I_1 in Fig. 9.18 (applies to $I''_1 + I'''_1$)

Opposite direction to I_1 in Fig. 9.18 (applies to I'_1)

$$I_1 = \underbrace{I''_1 + I'''_1} - I'_1$$
$$= 1\text{ A} + 2\text{ A} - 2\text{ A} = \mathbf{1\ A}$$

For the circuit in Fig. 4.12, use the superposition theorem to find i.

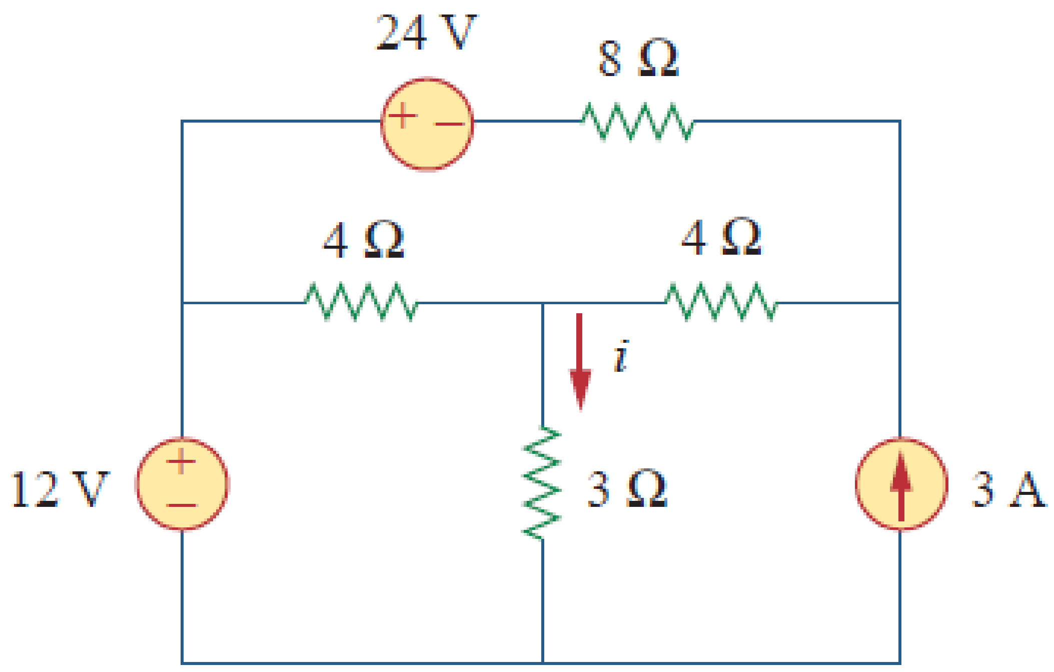


Figure 4.12
For Example 4.5.

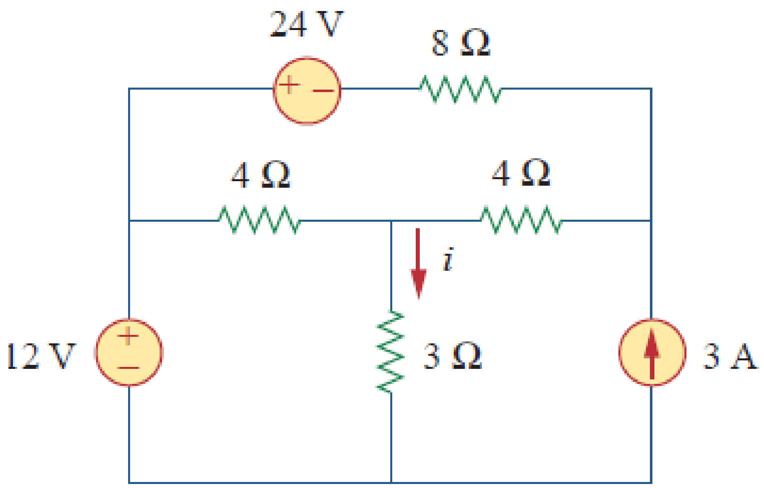
Solution:

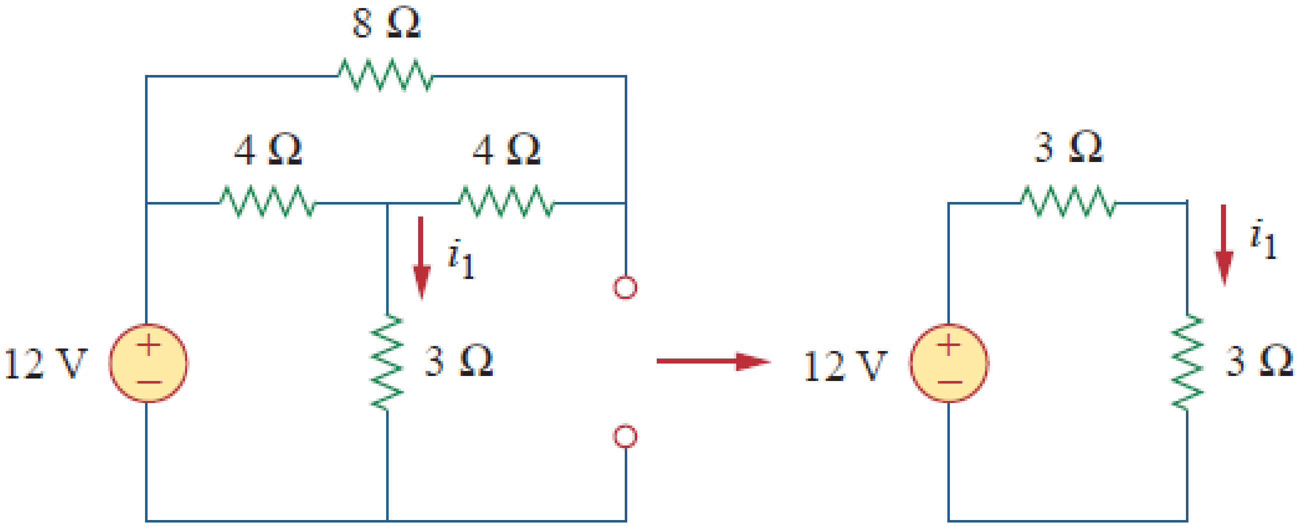
In this case, we have three sources. Let

$$i = i_1 + i_2 + i_3$$

where i_1, i_2, and i_3 are due to the 12-V, 24-V, and 3-A sources respectively. To get i_1, consider the circuit in Fig. 4.13(a). Combining 4 Ω (on the right-hand side) in series with 8 Ω gives 12 Ω. The 12 Ω in parallel with 4 Ω gives $12 \times 4/16 = 3$ Ω. Thus,

$$i_1 = \frac{12}{6} = 2 \text{ A}$$





(a)

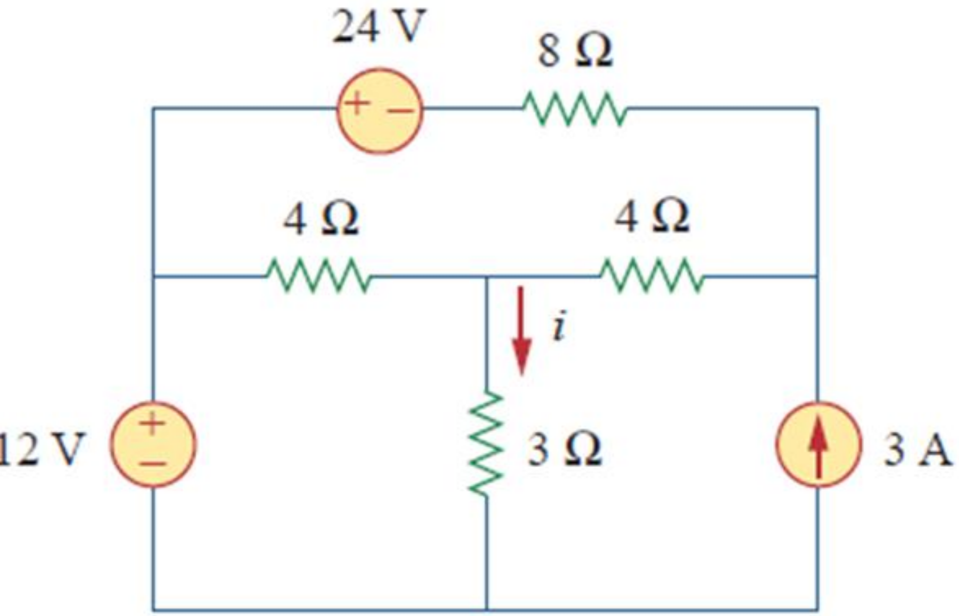
To get i_2, consider the circuit in Fig. 4.13(b). Applying mesh analysis gives

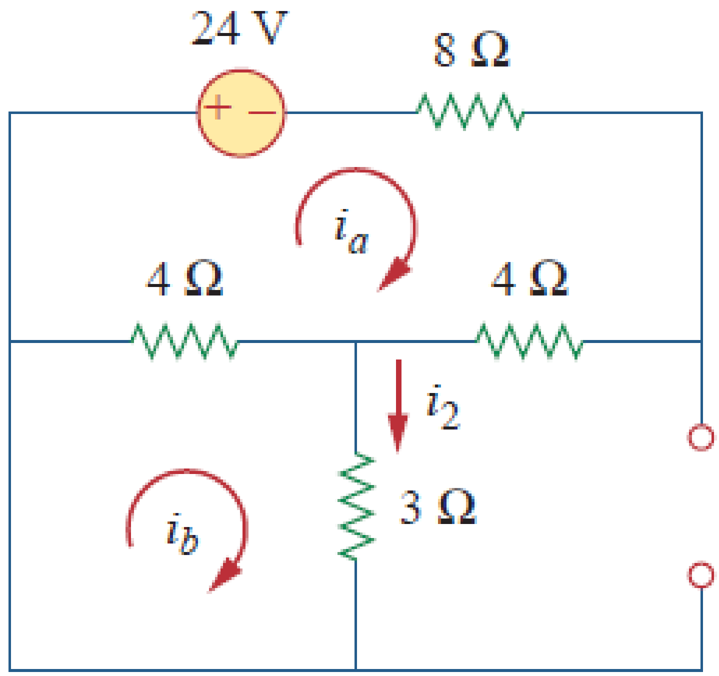
$$16i_a - 4i_b + 24 = 0 \quad \Rightarrow \quad 4i_a - i_b = -6 \qquad \textbf{(4.5.1)}$$

$$7i_b - 4i_a = 0 \quad \Rightarrow \quad i_a = \frac{7}{4}i_b \qquad \textbf{(4.5.2)}$$

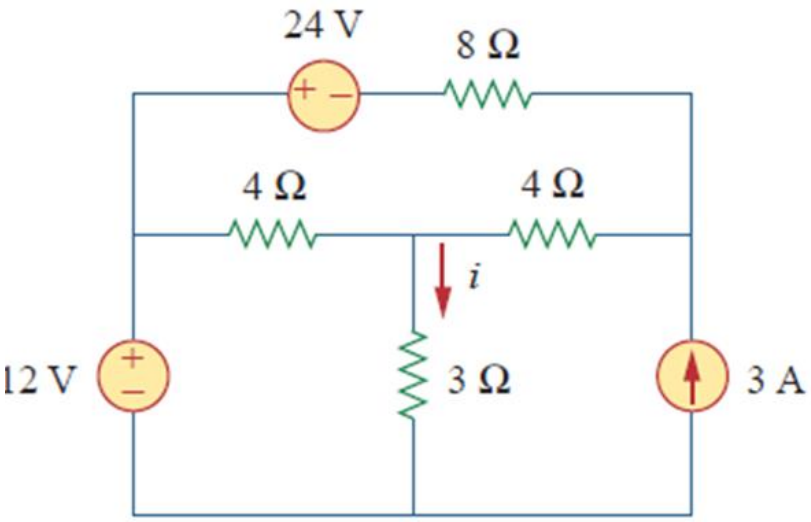
Substituting Eq. (4.5.2) into Eq. (4.5.1) gives

$$i_2 = i_b = -1$$





To get i_3, consider the circuit in Fig. 4.13(c). Using nodal analysis gives

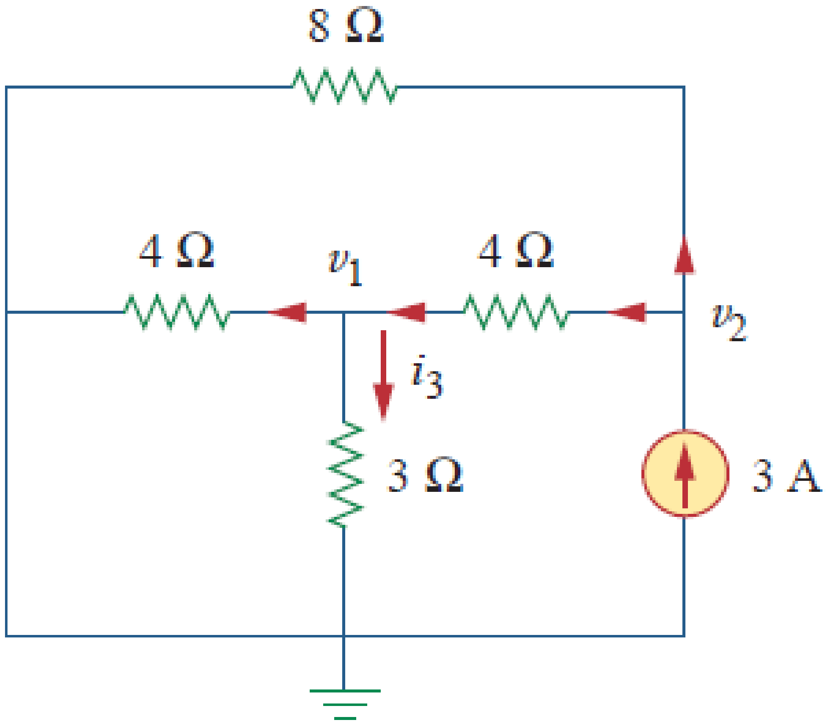


$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \quad \Rightarrow \quad 24 = 3v_2 - 2v_1 \qquad \textbf{(4.5.3)}$$

$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \quad \Rightarrow \quad v_2 = \frac{10}{3}v_1 \qquad \textbf{(4.5.4)}$$

Substituting Eq. (4.5.4) into Eq. (4.5.3) leads to $v_1 = 3$ and

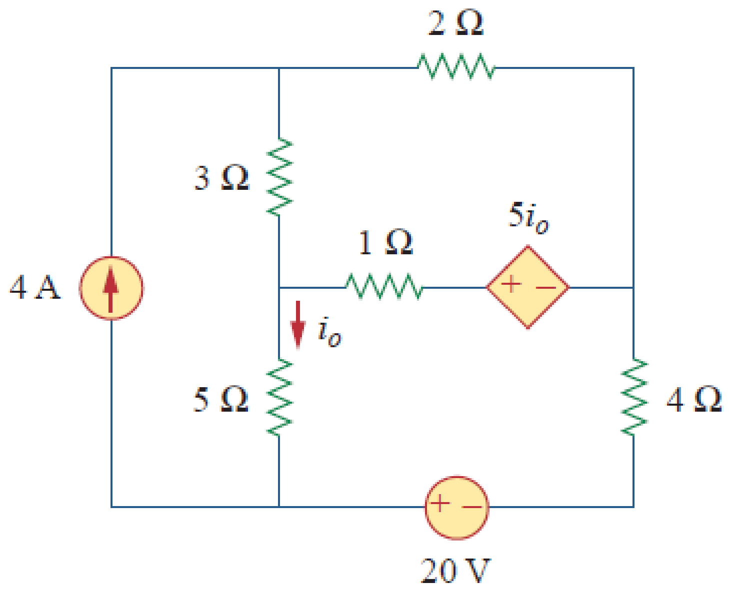
$$i_3 = \frac{v_1}{3} = 1 \text{ A}$$



Thus,

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2 \text{ A}$$

Find i_o in the circuit of Fig. 4.9 using superposition.



Solution:

The circuit in Fig. 4.9 involves a dependent source, which must be left intact. We let

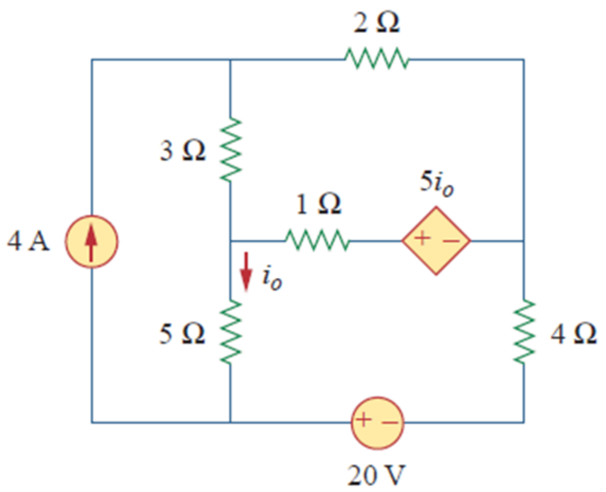
$$i_o = i'_o + i''_o \tag{4.4.1}$$

where i'_o and i''_o are due to the 4-A current source and 20-V voltage source respectively. To obtain i'_o, we turn off the 20-V source so that we have the circuit in Fig. 4.10(a). We apply mesh analysis in order to obtain i'_o. For loop 1,

$$i_1 = 4 \text{ A} \tag{4.4.2}$$

For loop 2,

$$-3i_1 + 6i_2 - 1i_3 - 5i'_o = 0 \tag{4.4.3}$$



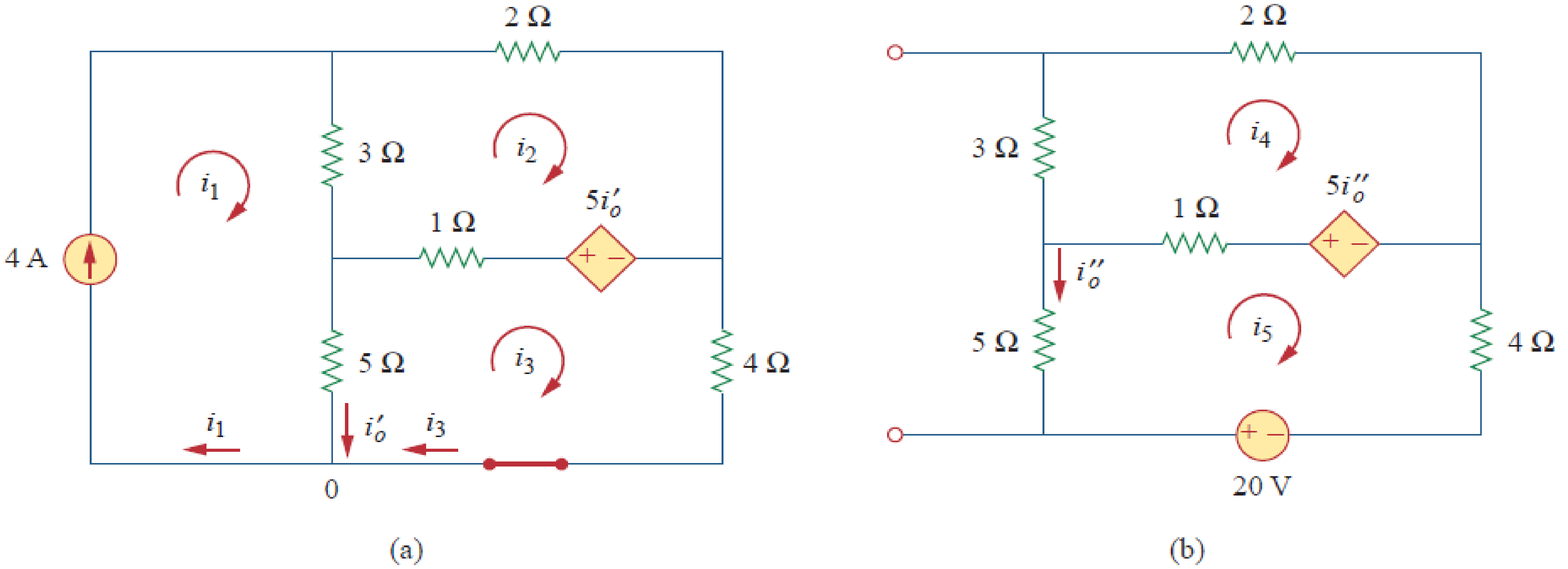


Figure 4.10
For Example 4.4: Applying superposition to (a) obtain i'_o, (b) obtain i''_o.

9.3 THÉVENIN'S THEOREM

The next theorem to be introduced, **Thévenin's theorem,** is probably one of the most interesting in that it permits the reduction of complex networks to a simpler form for analysis and design.

In general, the theorem can be used to do the following:

- *Analyze networks with sources that are not in series or parallel.*
- ***Reduce the number of components required to establish the same characteristics at the output terminals.***
- ***Investigate the effect of changing a particular component on the behavior of a network without having to analyze the entire network after each change.***

All three areas of application are demonstrated in the examples to follow.

Thévenin's theorem states the following:

Any two-terminal dc network can be replaced by an equivalent circuit consisting solely of a voltage source and a series resistor as shown in Fig. 9.23.

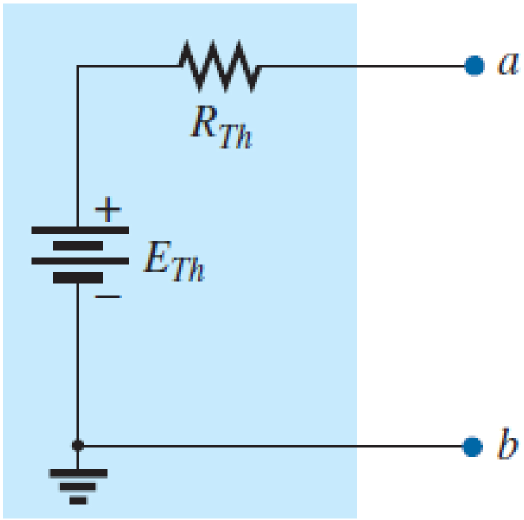


FIG. 9.23
Thévenin equivalent circuit.

To demonstrate the power of the theorem, consider the fairly complex network of Fig. 9.25(a) with its two sources and series-parallel connections. The theorem states that the entire network inside the blue shaded area can be replaced by one voltage source and one resistor as shown in Fig. 9.25(b). If the replacement is done properly, the voltage across, and the current through, the resistor R_L will be the same for each network. The value of R_L can be changed to any value, and the voltage, current, or power to the load resistor is the same for each configuration. Now, this is a very powerful statement—one that is verified in the examples to follow.

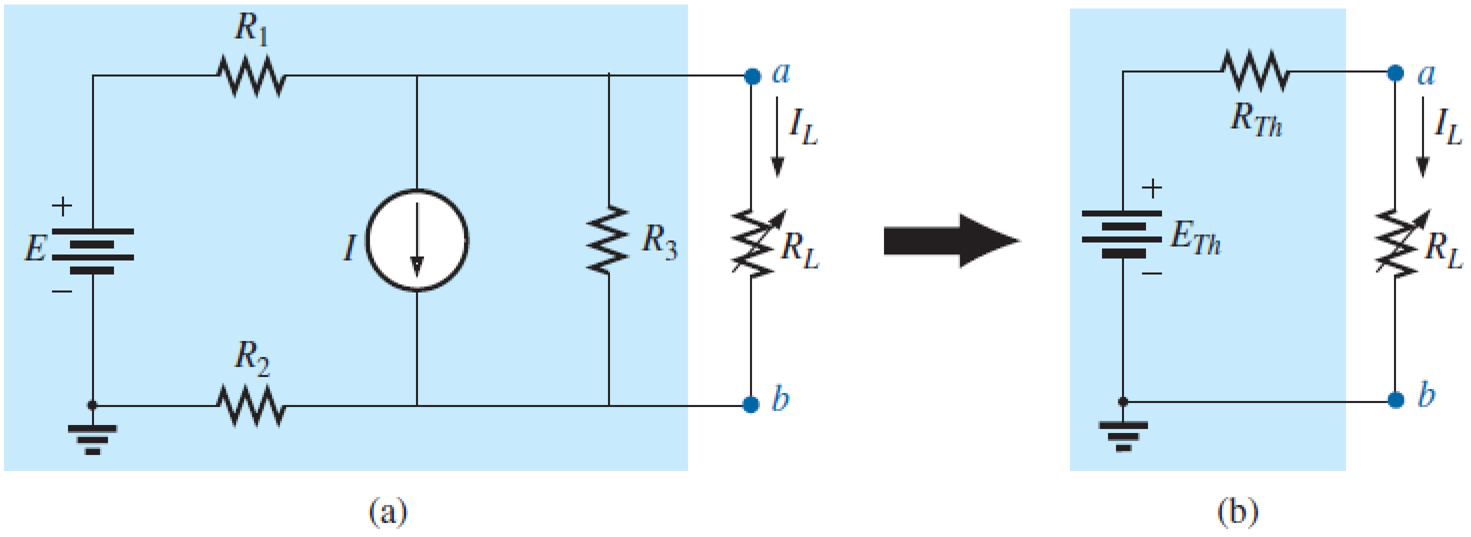


FIG. 9.25
Substituting the Thévenin equivalent circuit for a complex network.

The question then is, How can you determine the proper value of Thévenin voltage and resistance? In general, finding the Thévenin *resistance* value is quite straightforward. Finding the Thévenin *voltage* can be more of a challenge and, in fact, may require using the superposition theorem or one of the methods described in Chapter 8.

Thévenin's Theorem Procedure

Preliminary:

1. *Remove that portion of the network where the Thévenin equivalent circuit is found. In Fig. 9.25(a), this requires that the load resistor R_L be temporarily removed from the network.*
2. *Mark the terminals of the remaining two-terminal network. (The importance of this step will become obvious as we progress through some complex networks.)*

R_{Th}:

3. ***Calculate R_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)***

E_{Th}:

4. ***Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (This step is invariably the one that causes most confusion and errors. In all cases, keep in mind that it is the open-circuit potential between the two terminals marked in step 2.)***

Conclusion:

5. ***Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit. This step is indicated by the placement of the resistor R_L between the terminals of the Thévenin equivalent circuit as shown in Fig. 9.25(b).***

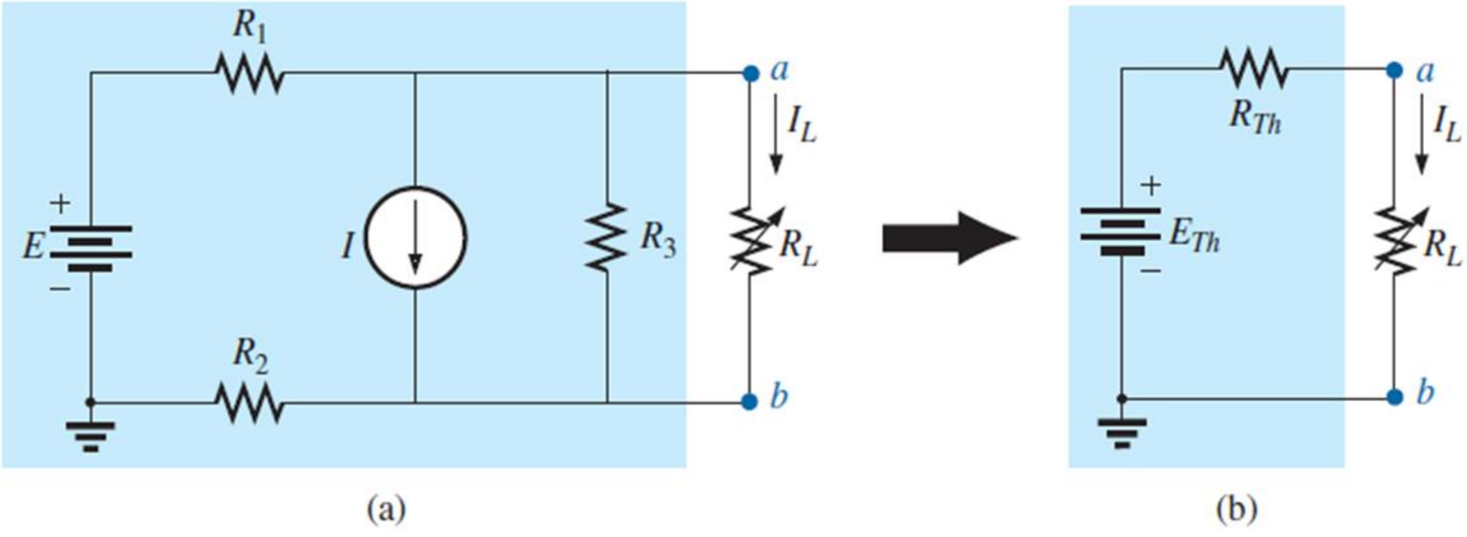


FIG. 9.25
Substituting the Thévenin equivalent circuit for a complex network.

EXAMPLE 9.6 Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig. 9.26. Then find the current through R_L for values of 2 Ω, 10 Ω, and 100 Ω.

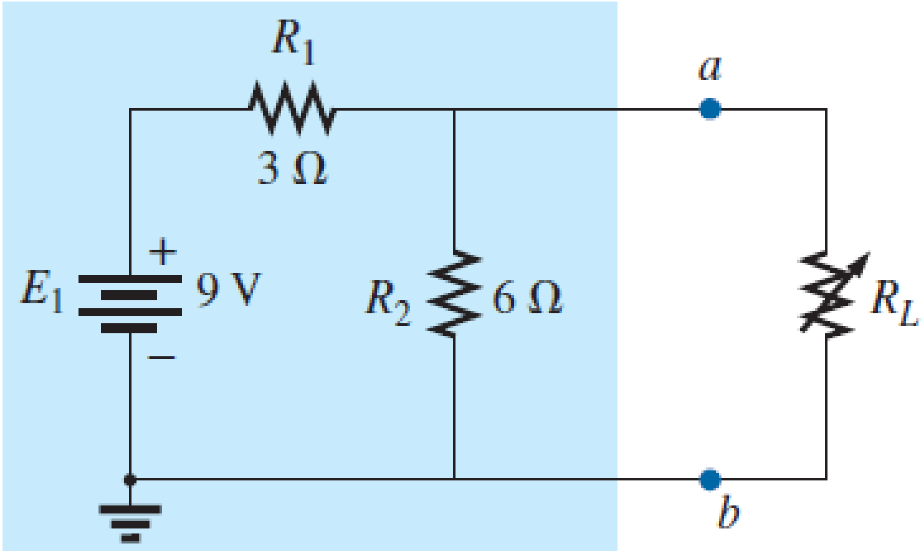


FIG. 9.26
Example 9.6.

Solution:

Steps 1 and 2: These produce the network in Fig. 9.27. Note that the load resistor R_L has been removed and the two "holding" terminals have been defined as a and b.

Steps 3: Replacing the voltage source E_1 with a short-circuit equivalent yields the network in Fig. 9.28(a), where

$$R_{Th} = R_1 \| R_2 = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = 2\ \Omega$$

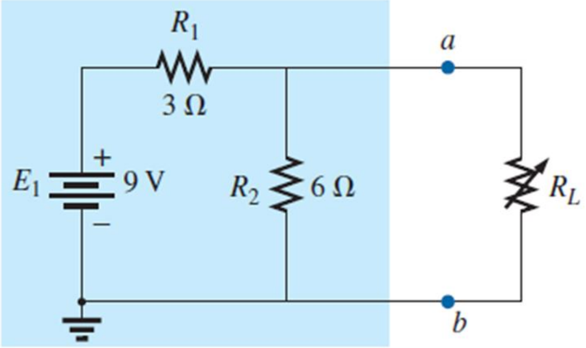
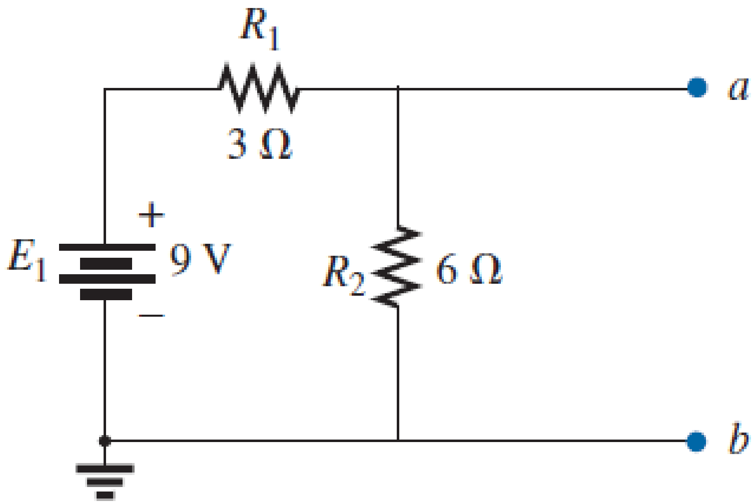
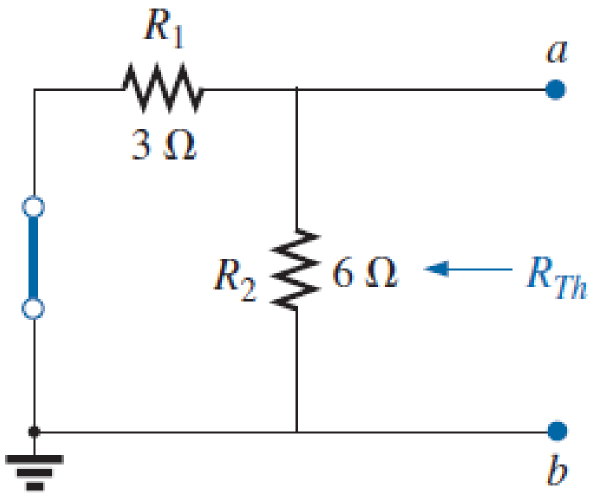
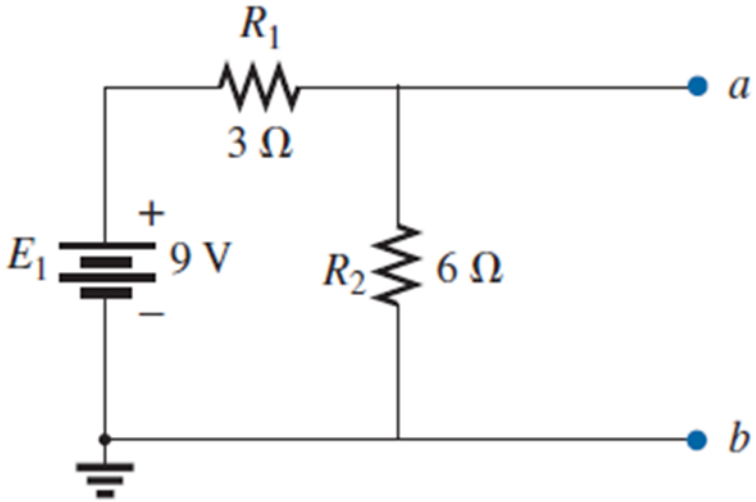


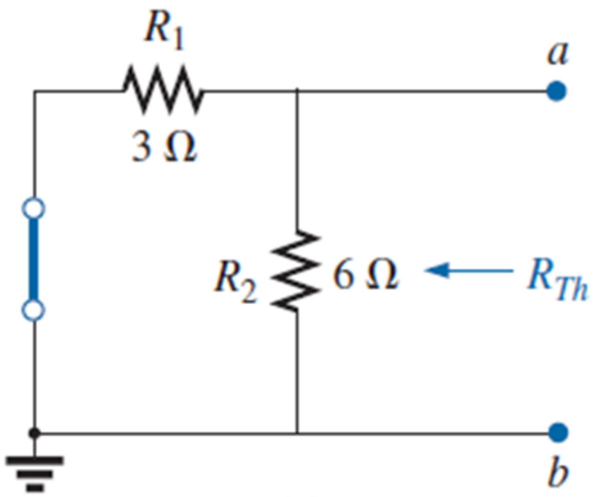
FIG. 9.26
Example 9.6.



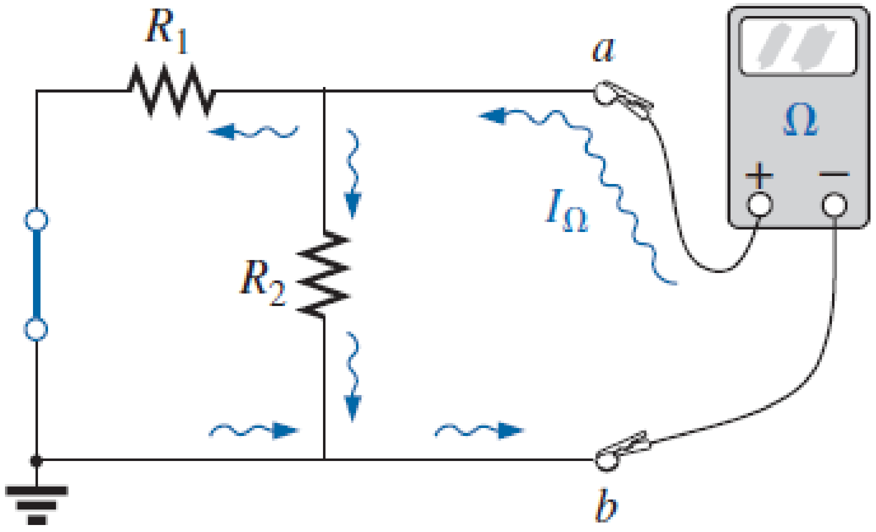


(a)





(a)



(b)

Step 4: Replace the voltage source (Fig. 9.29). For this case, the open-circuit voltage E_{Th} is the same as the voltage drop across the 6 Ω resistor. Applying the voltage divider rule,

$$E_{Th} = \frac{R_2E_1}{R_2 + R_1} = \frac{(6\ \Omega)(9\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{54\ \text{V}}{9} = \mathbf{6\ V}$$

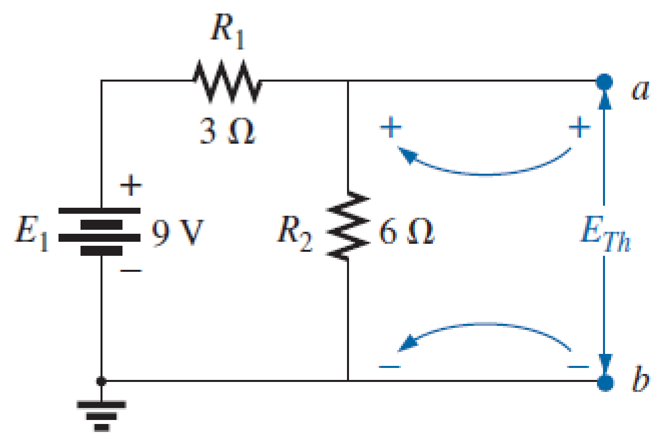


FIG. 9.29
Determining E_{Th} for the network in Fig. 9.27.

It is particularly important to recognize that E_{Th} is the open-circuit potential between points *a* and *b*. Remember that an open circuit can have any voltage across it, but the current must be zero. In fact, the current through any element in series with the open circuit must be zero also. The use of a voltmeter to measure E_{Th} appears in Fig. 9.30. Note that it is placed directly across the resistor R_2 since E_{Th} and V_{R_2} are in parallel.

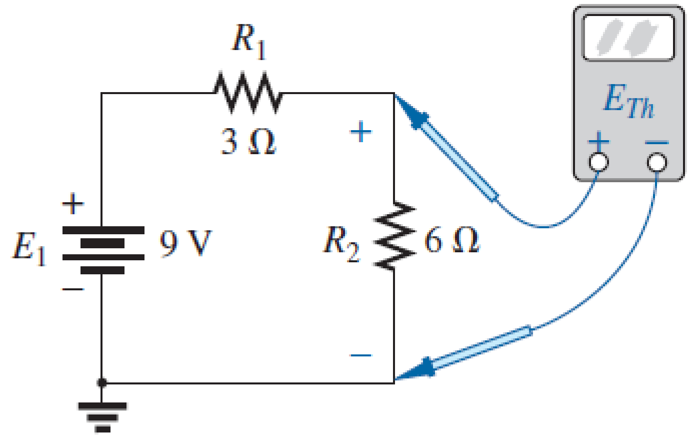


FIG. 9.30

Step 5 (Fig. 9.31):

$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$

$$R_L = 2\ \Omega: \qquad I_L = \frac{6\ \text{V}}{2\ \Omega + 2\ \Omega} = \mathbf{1.5\ A}$$

$$R_L = 10\ \Omega: \qquad I_L = \frac{6\ \text{V}}{2\ \Omega + 10\ \Omega} = \mathbf{0.5\ A}$$

$$R_L = 100\ \Omega: \qquad I_L = \frac{6\ \text{V}}{2\ \Omega + 100\ \Omega} = \mathbf{0.06\ A}$$

If Thévenin's theorem were unavailable, each change in R_L would require that the entire network in Fig. 9.26 be reexamined to find the new value of R_L.

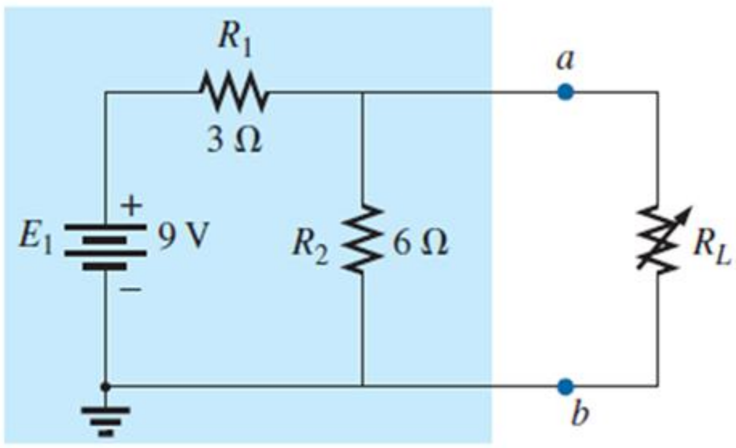


FIG. 9.26
Example 9.6.

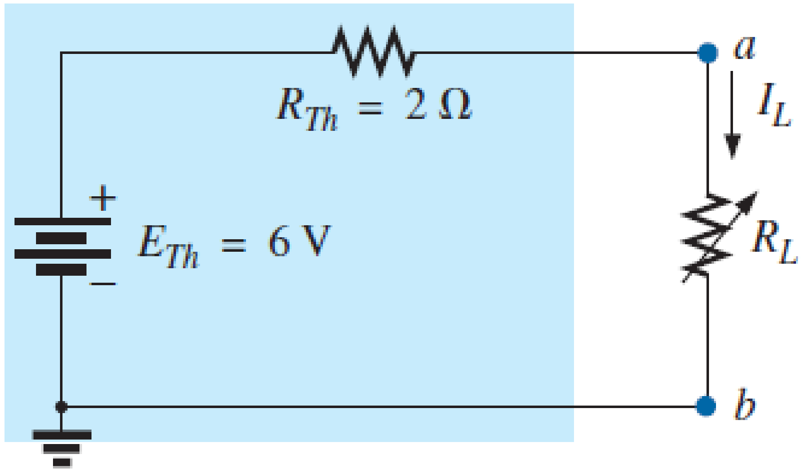


FIG. 9.31
Substituting the Thévenin equivalent circuit for the network external to R_L in Fig. 9.26.

EXAMPLE 9.7 Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig. 9.32.

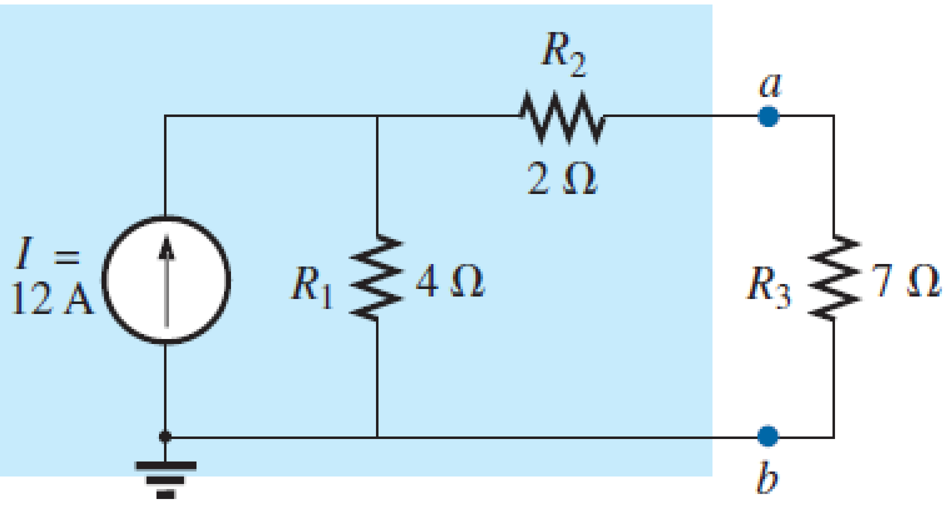
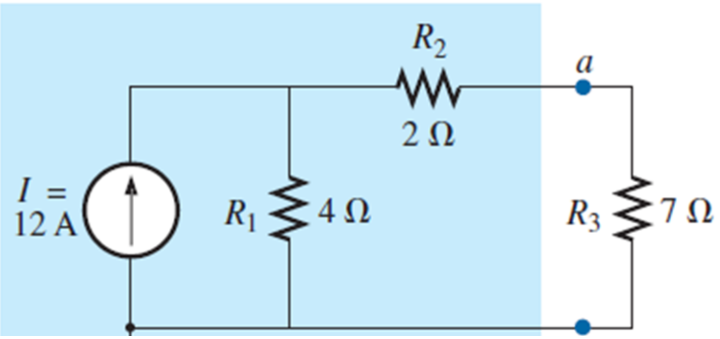


FIG. 9.32
Example 9.7.



Solution:

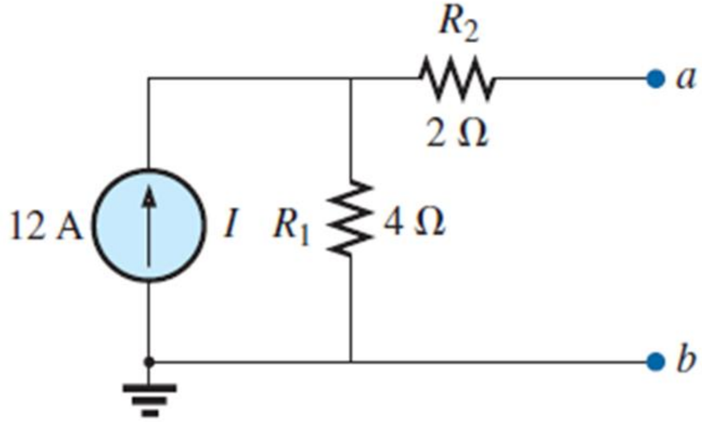


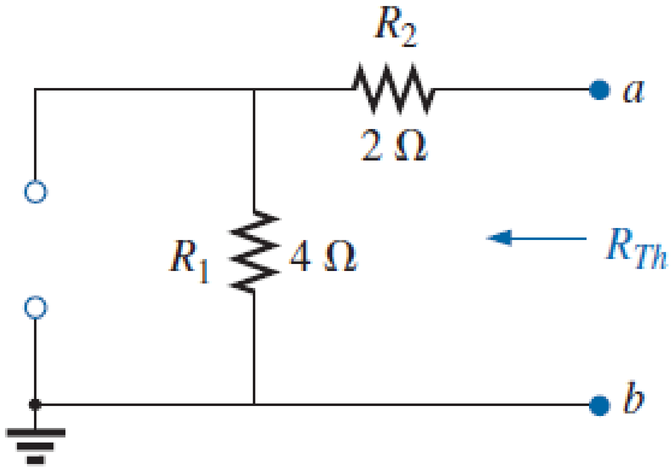
FIG. 9.33

Steps 1 and 2: See Fig. 9.33.

Step 3: See Fig. 9.34. The current source has been replaced with an open-circuit equivalent, and the resistance determined between terminals a and b.

In this case, an ohmmeter connected between terminals a and b sends out a sensing current that flows directly through R_1 and R_2 (at the same level). The result is that R_1 and R_2 are in series and the Thévenin resistance is the sum of the two.

$$R_{Th} = R_1 + R_2 = 4\ \Omega + 2\ \Omega = \mathbf{6\ \Omega}$$



Step 4: See Fig. 9.35. In this case, since an open circuit exists between the two marked terminals, the current is zero between these terminals and through the 2 Ω resistor. The voltage drop across R_2 is, therefore,

$$V_2 = I_2R_2 = (0)R_2 = 0\text{ V}$$

and
$$E_{Th} = V_1 = I_1R_1 = IR_1 = (12\text{ A})(4\ \Omega) = \mathbf{48\ V}$$

Step 5: See Fig. 9.36.

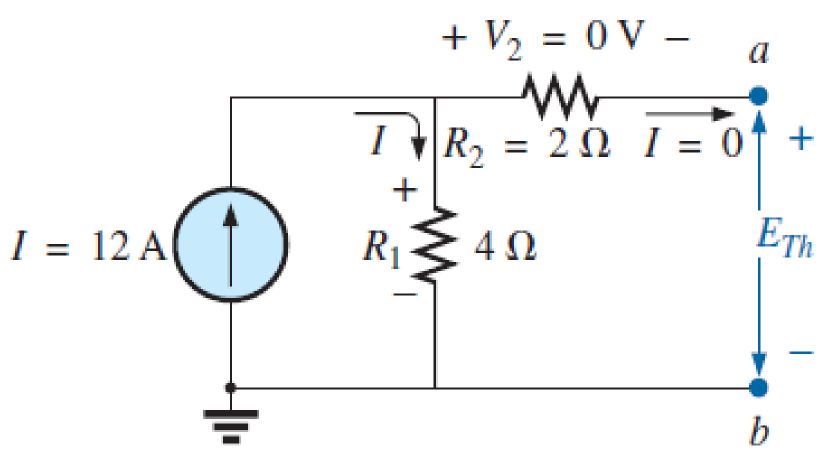


FIG. 9.35
Determining E_{Th} for the network in Fig. 9.33.

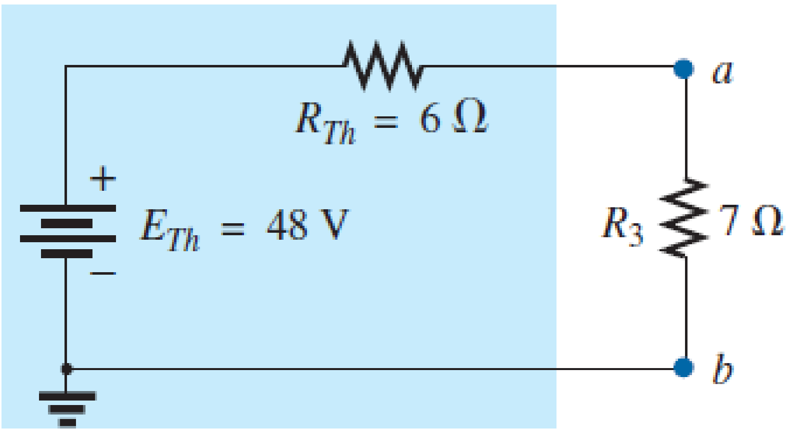
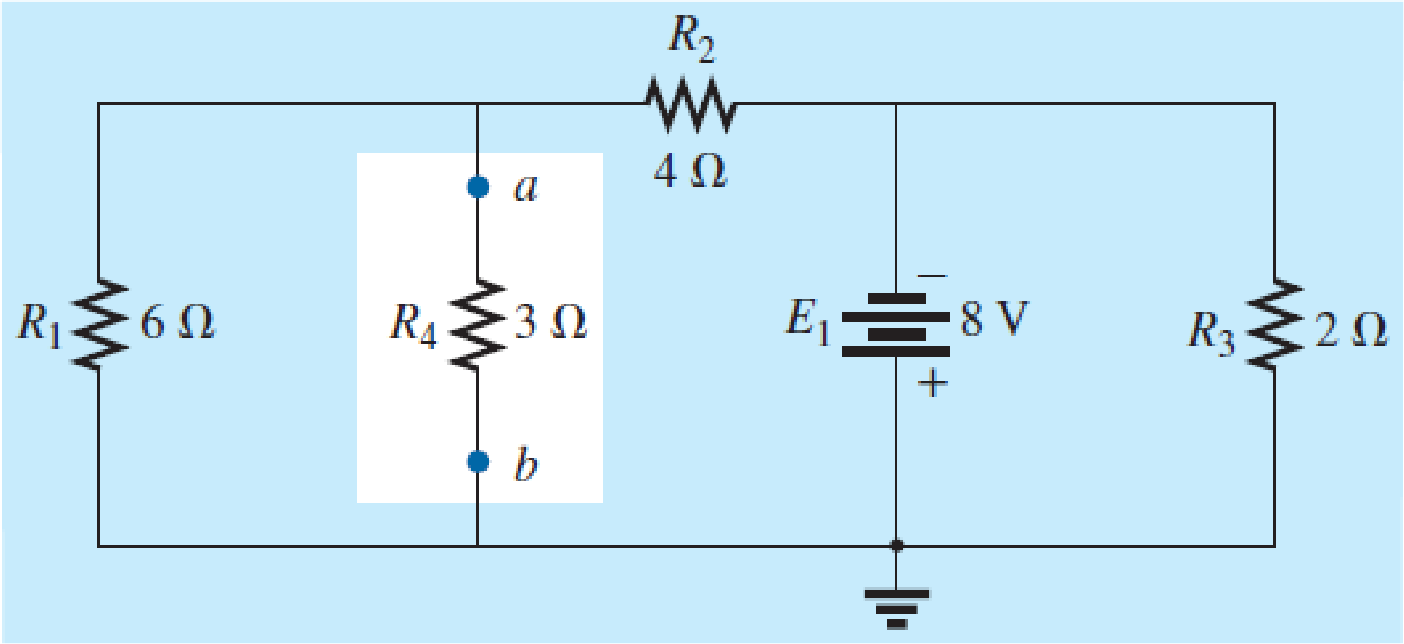


FIG. 9.36
Substituting the Thévenin equivalent circuit in the network external to the resistor R_3 in Fig. 9.32.

EXAMPLE 9.8 Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig. 9.37. Note in this example that there is no need for the section of the network to be preserved to be at the "end" of the configuration.



Solution:

Steps 1 and 2: See Fig. 9.38.

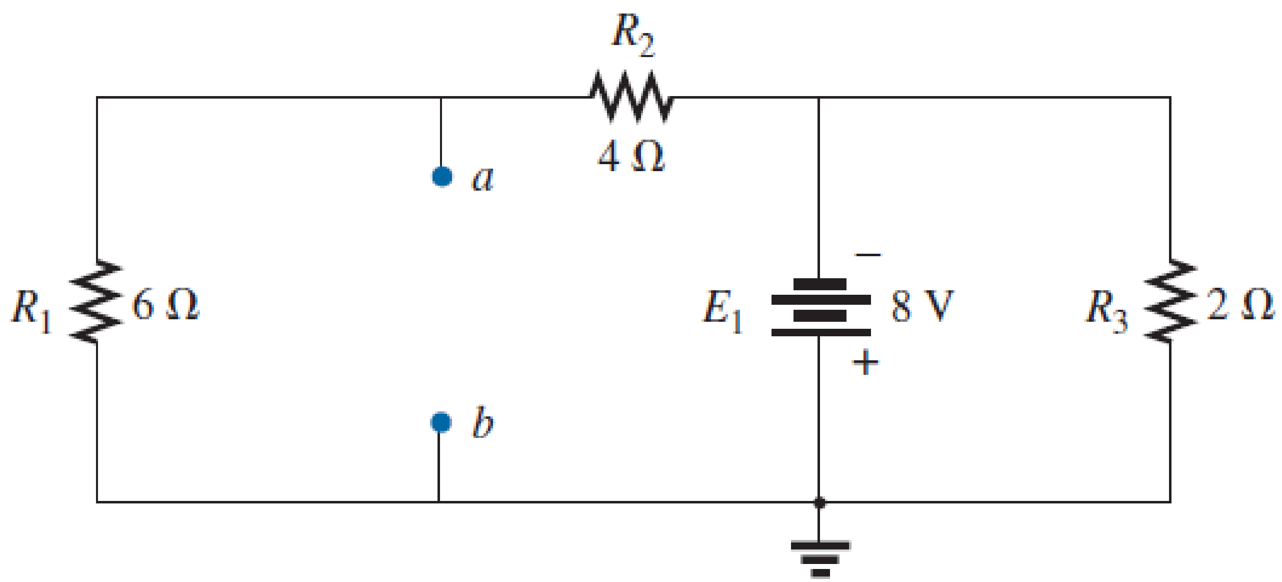


FIG. 9.38

Identifying the terminals of particular interest for the network in Fig. 9.37.

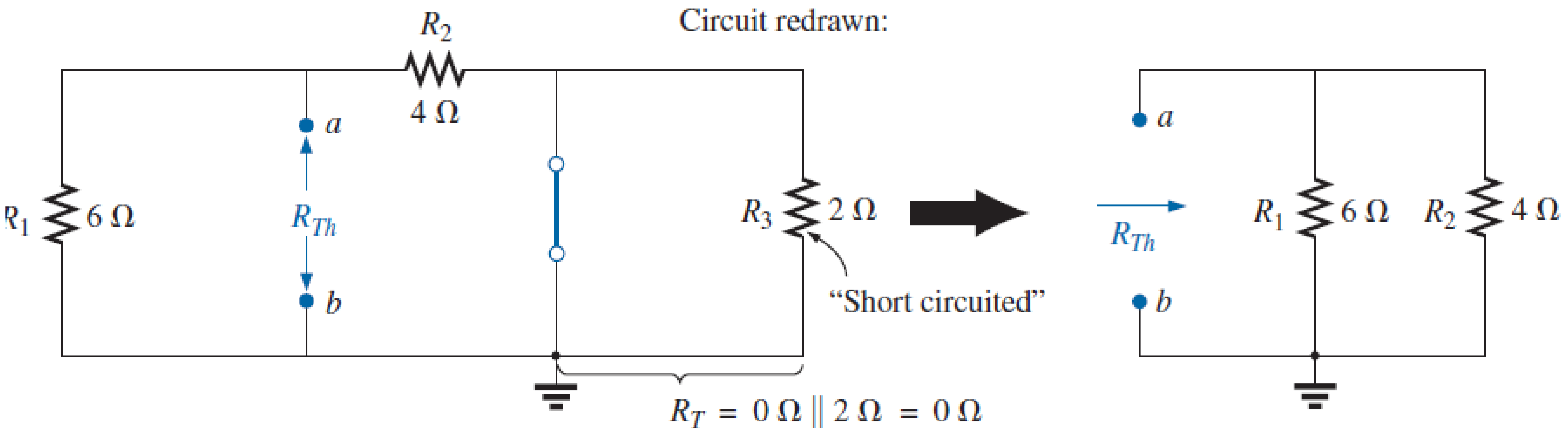


FIG. 9.39

Determining R_{Th} for the network in Fig. 9.38.

$$R_{Th} = R_1 \| R_2 = \frac{(6\ \Omega)(4\ \Omega)}{6\ \Omega + 4\ \Omega} = \frac{24\ \Omega}{10} = \mathbf{2.4\ \Omega}$$

Step 4: See Fig. 9.40. In this case, the network can be redrawn as shown in Fig. 9.41. Since the voltage is the same across parallel elements, the voltage across the series resistors R_1 and R_2 is E_1, or 8 V. Applying the voltage divider rule,

$$E_{Th} = \frac{R_1 E_1}{R_1 + R_2} = \frac{(6\ \Omega)(8\ \text{V})}{6\ \Omega + 4\ \Omega} = \frac{48\ \text{V}}{10} = \mathbf{4.8\ V}$$

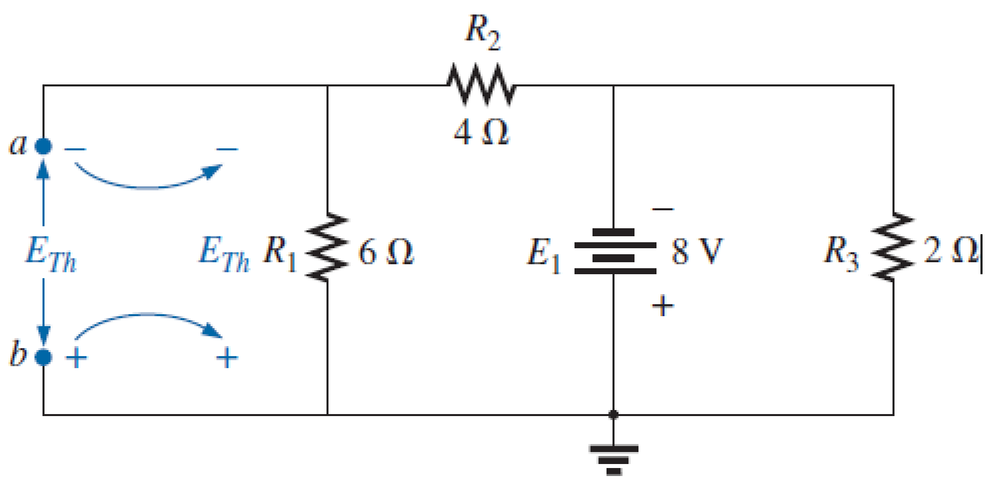


FIG. 9.40
Determining E_{Th} for the network in Fig. 9.38.

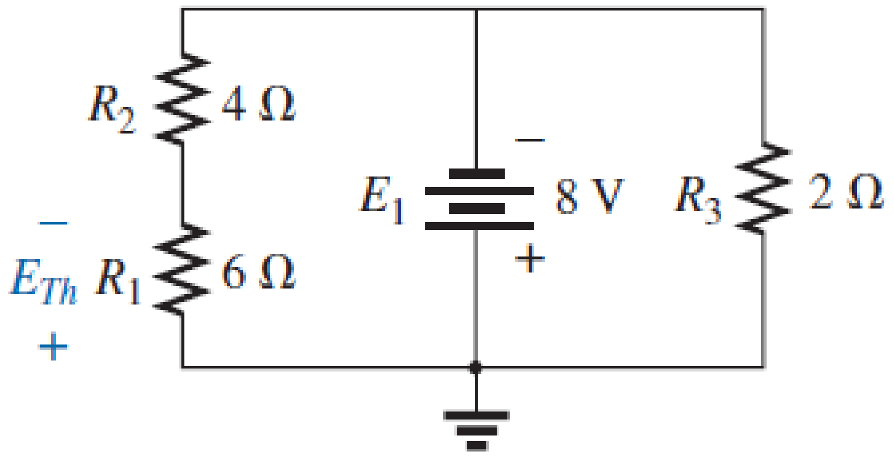
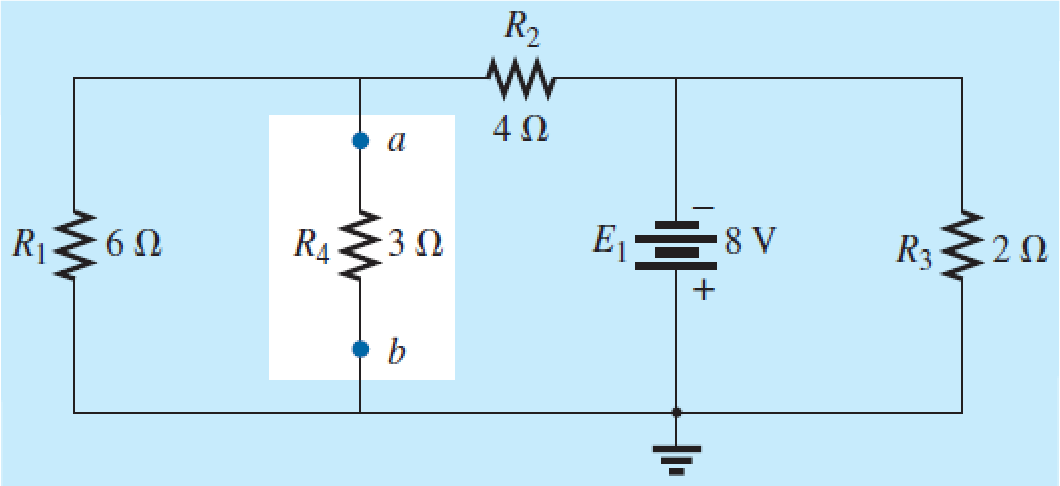


FIG. 9.41
Network of Fig. 9.40 redrawn.



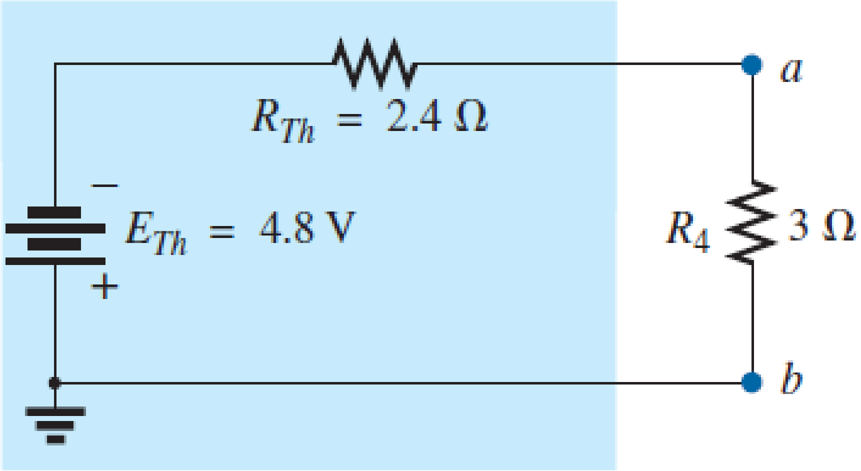


FIG. 9.42

Substituting the Thévenin equivalent circuit for the network external to the resistor R_4 in Fig. 9.37.

EXAMPLE 9.9 Find the Thévenin equivalent circuit for the network in the shaded area of the bridge network in Fig. 9.43.

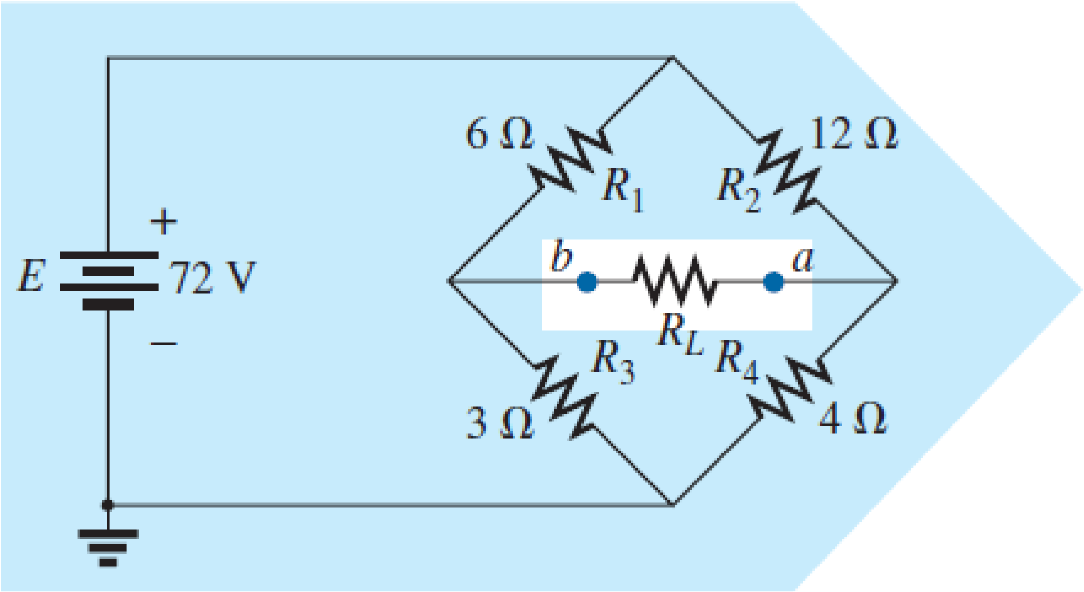


FIG. 9.43
Example 9.9.

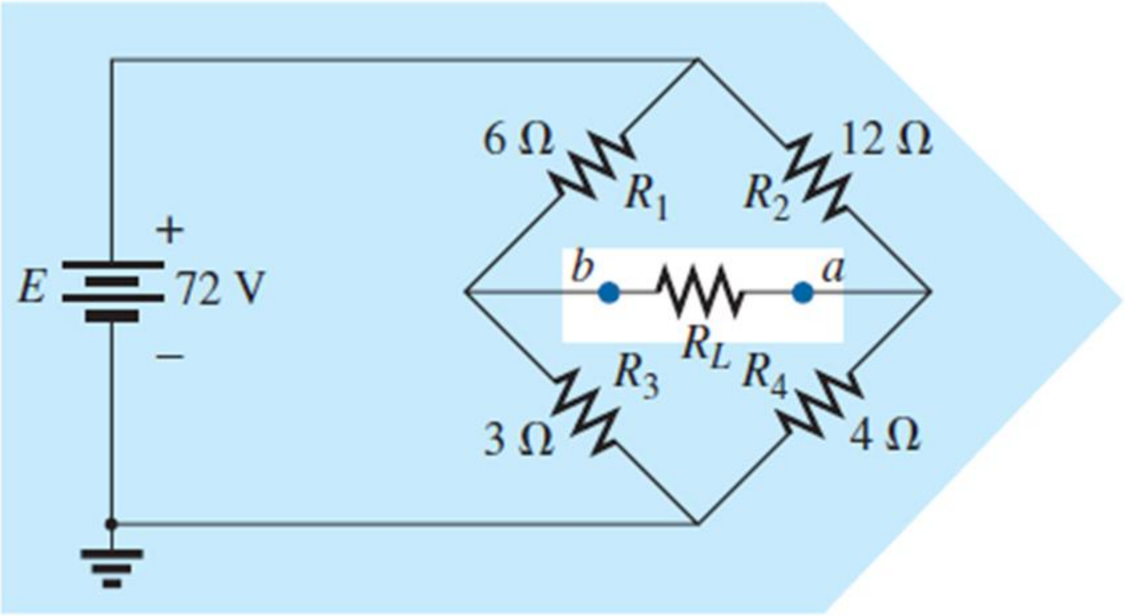
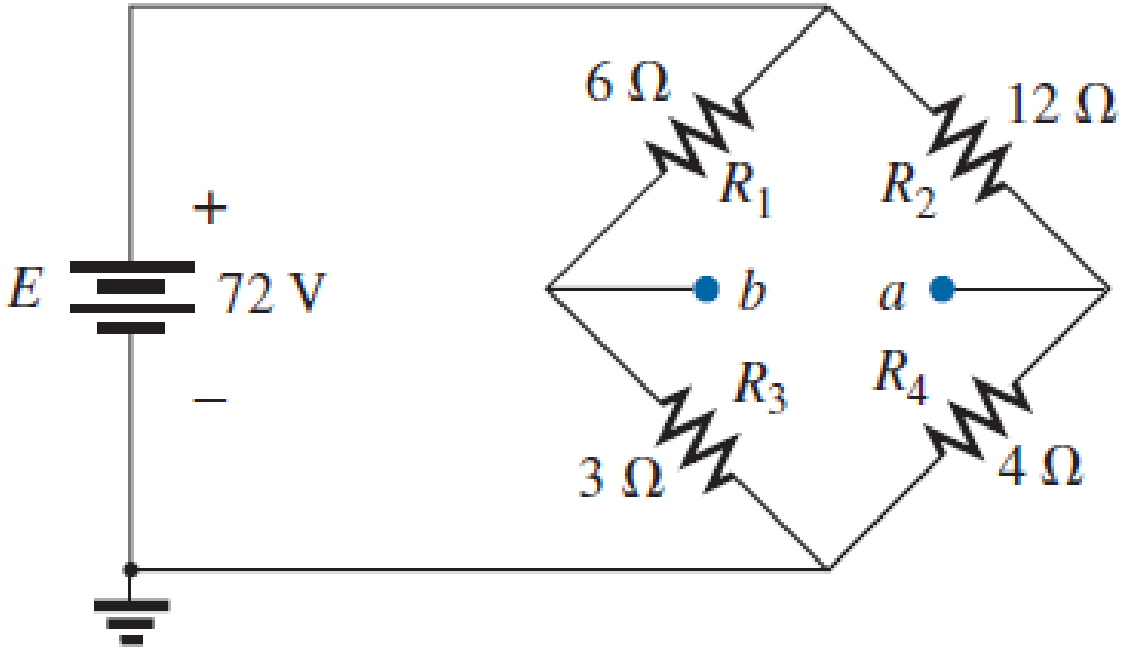


FIG. 9.43
Example 9.9.



Step 3: See Fig. 9.45. In this case, the short-circuit replacement of the voltage source E provides a direct connection between c and c' in Fig. 9.45(a), permitting a "folding" of the network around the horizontal line of a-b to produce the configuration in Fig. 9.45(b).

$$R_{Th} = R_{a-b} = R_1 \parallel R_3 + R_2 \parallel R_4$$
$$= 6\ \Omega \parallel 3\ \Omega + 4\ \Omega \parallel 12\ \Omega$$
$$= 2\ \Omega + 3\ \Omega = \mathbf{5\ \Omega}$$

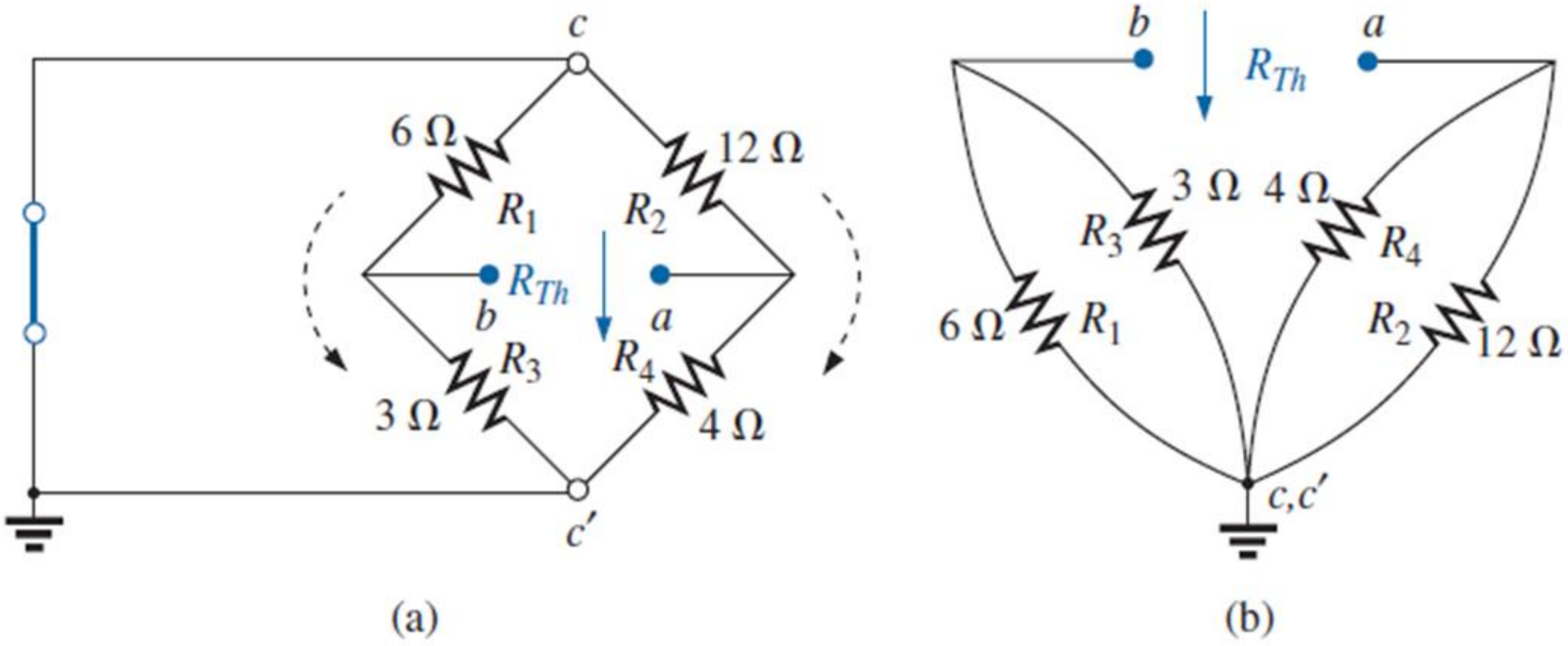


FIG. 9.45

Step 4: The circuit is redrawn in Fig. 9.46. The absence of a direct connection between *a* and *b* results in a network with three parallel branches. The voltages V_1 and V_2 can therefore be determined using the voltage divider rule:

$$V_1 = \frac{R_1 E}{R_1 + R_3} = \frac{(6\ \Omega)(72\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{432\ \text{V}}{9} = 48\ \text{V}$$

$$V_2 = \frac{R_2 E}{R_2 + R_4} = \frac{(12\ \Omega)(72\ \text{V})}{12\ \Omega + 4\ \Omega} = \frac{864\ \text{V}}{16} = 54\ \text{V}$$

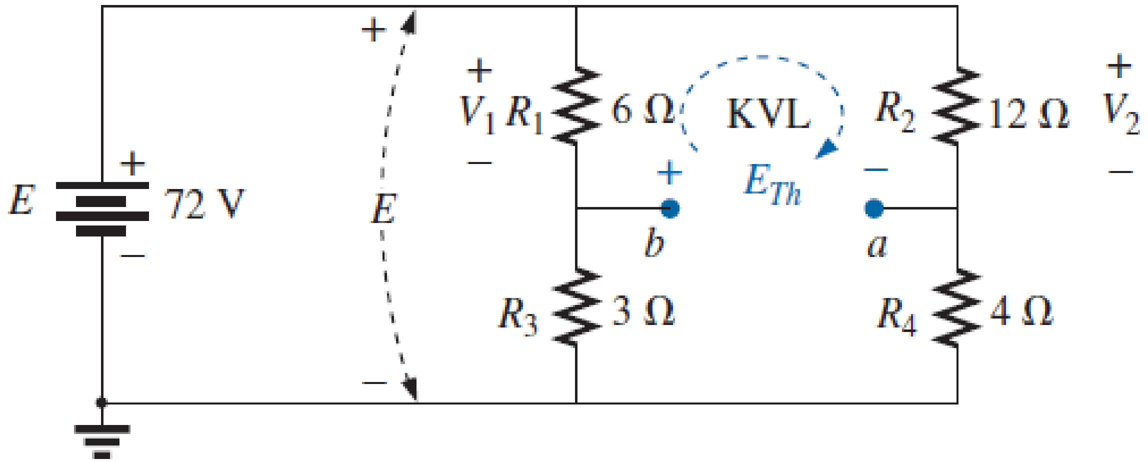


FIG. 9.46

Determining E_{Th} for the network in Fig. 9.44.

Assuming the polarity shown for E_{Th} and applying Kirchhoff's voltage law to the top loop in the clockwise direction results in

$$\Sigma_{\circlearrowright} V = +E_{Th} + V_1 - V_2 = 0$$

and

$$E_{Th} = V_2 - V_1 = 54\text{ V} - 48\text{ V} = \mathbf{6\ V}$$

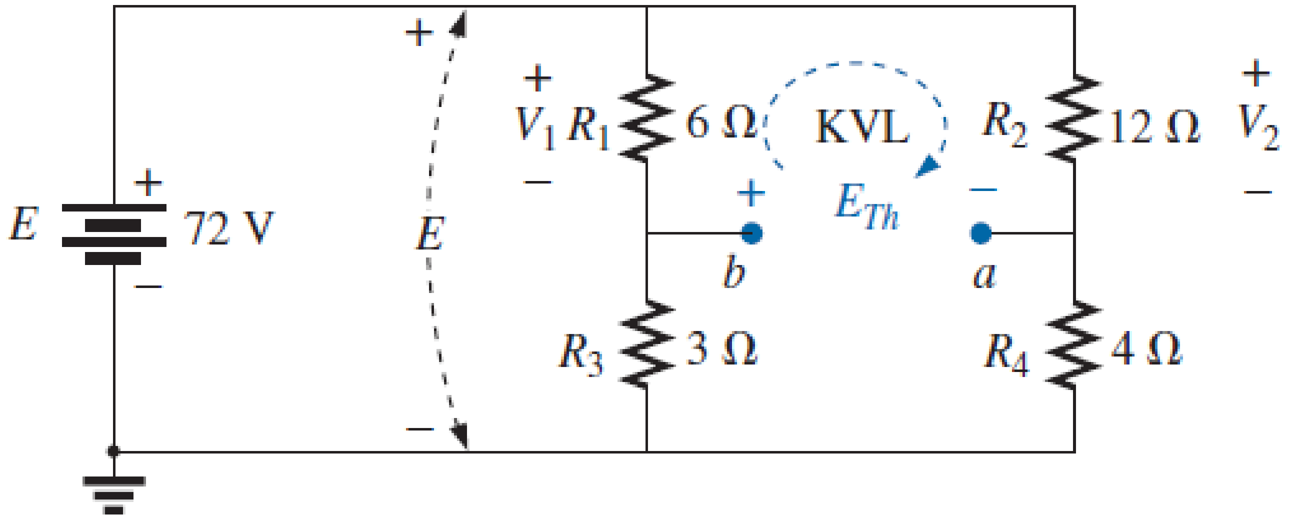


FIG. 9.46

Determining E_{Th} for the network in Fig. 9.44.

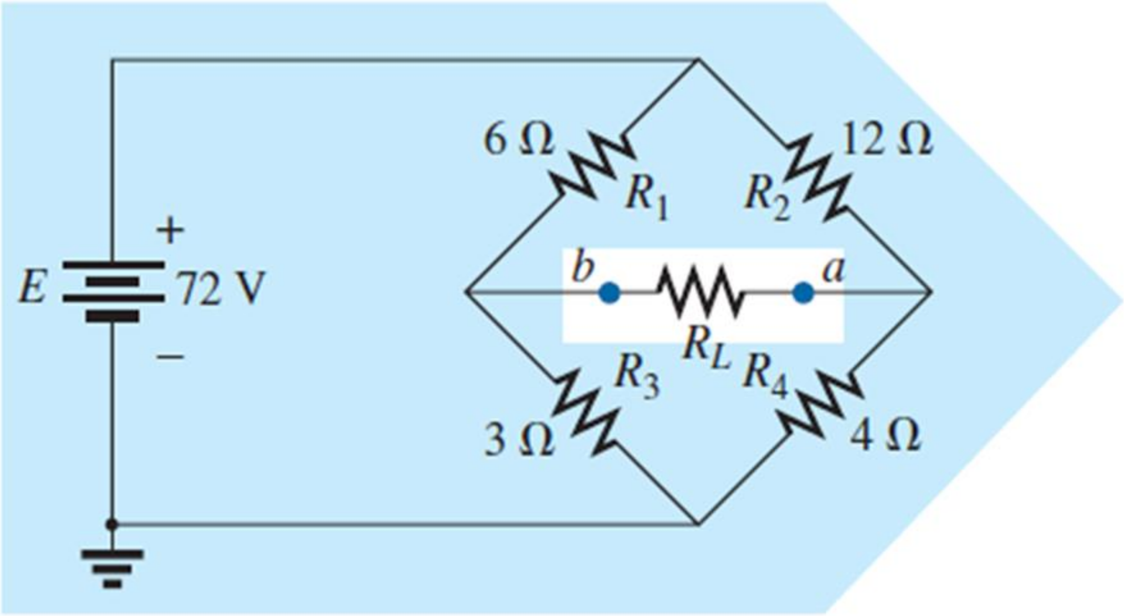


FIG. 9.43

Example 9.9.

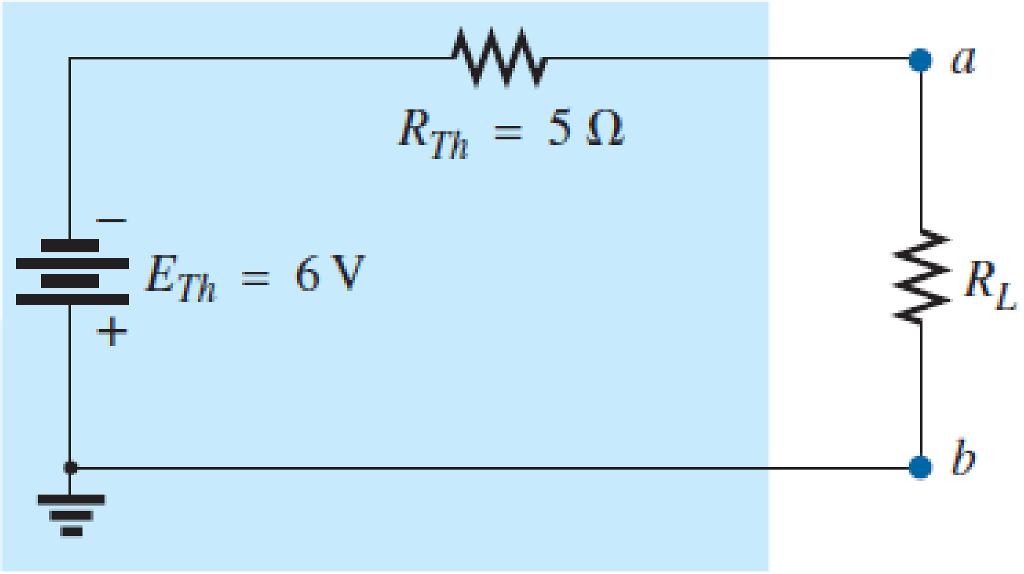


FIG. 9.47

Substituting the Thévenin equivalent circuit for the network external to the resistor R_L in Fig. 9.43.

Thévenin's theorem is not restricted to a single passive element, as shown in the preceding examples, but can be applied across sources, whole branches, portions of networks, or any circuit configuration as shown in the following example. It is also possible that you may have to use one of the methods previously described, such as mesh analysis or superposition, to find the Thévenin equivalent circuit.

EXAMPLE 9.10 (Two sources) Find the Thévenin circuit for the network within the shaded area of Fig. 9.48.

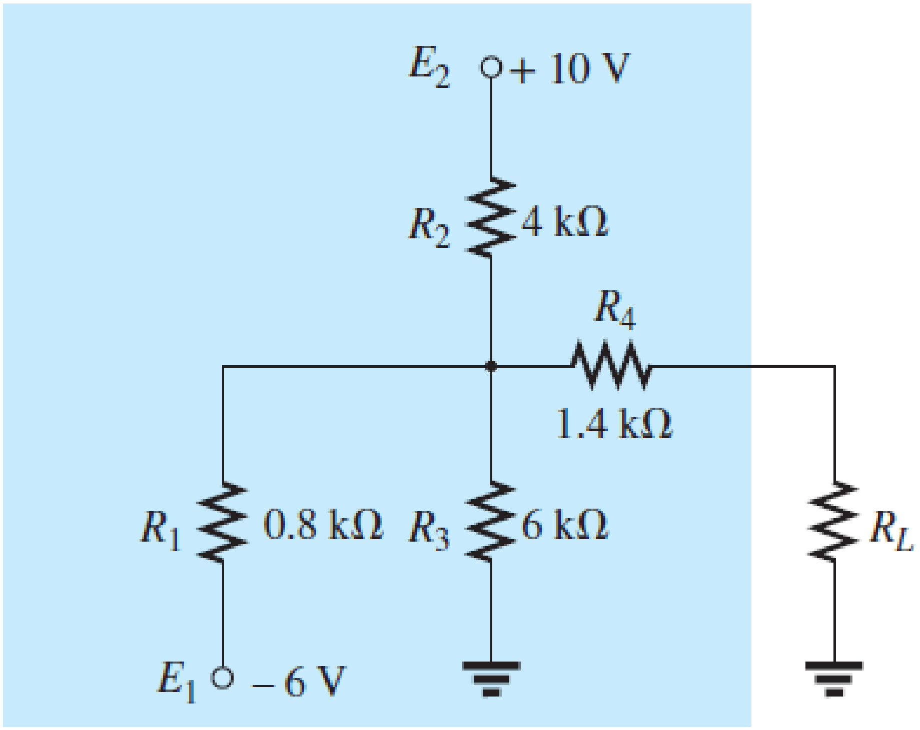


FIG. 9.48
Example 9.10.

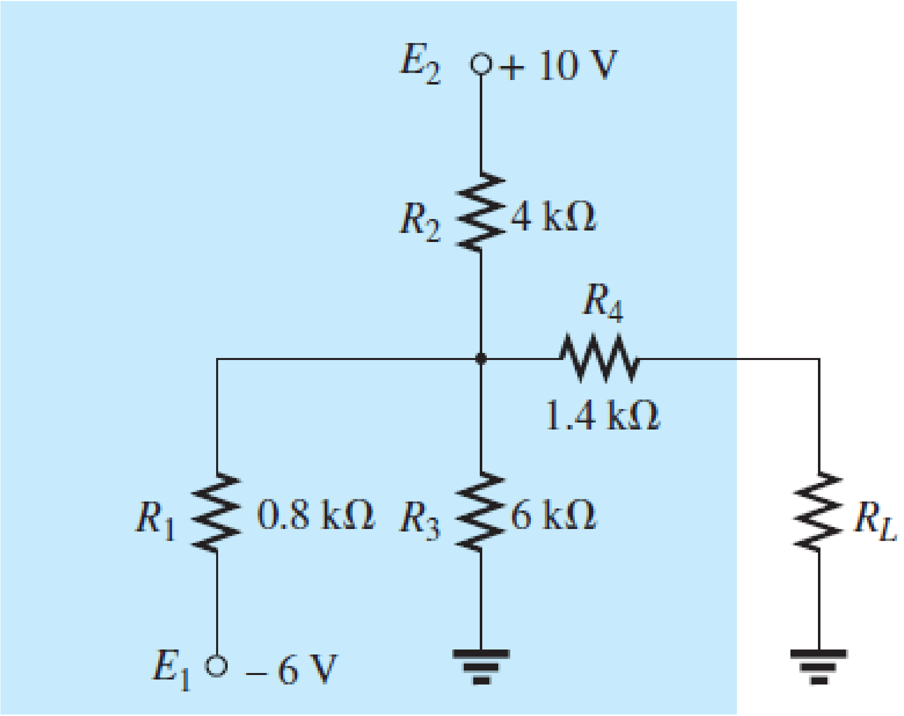
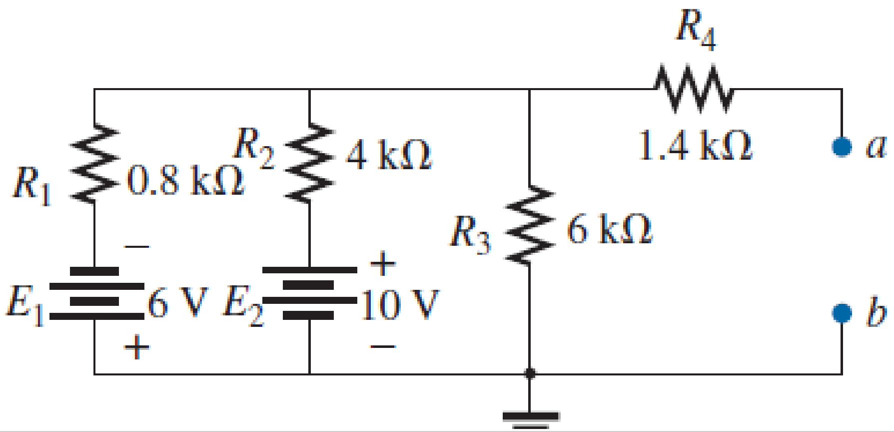


FIG. 9.48
Example 9.10.



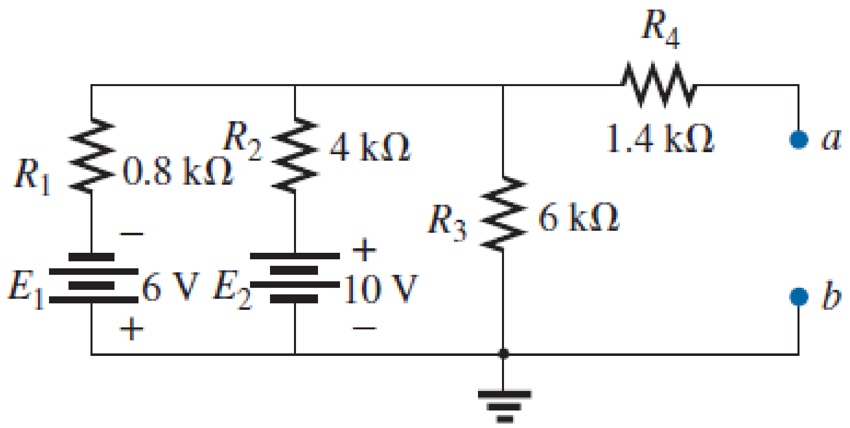


FIG. 9.49
Identifying the terminals of particular interest for the network in Fig. 9.48.

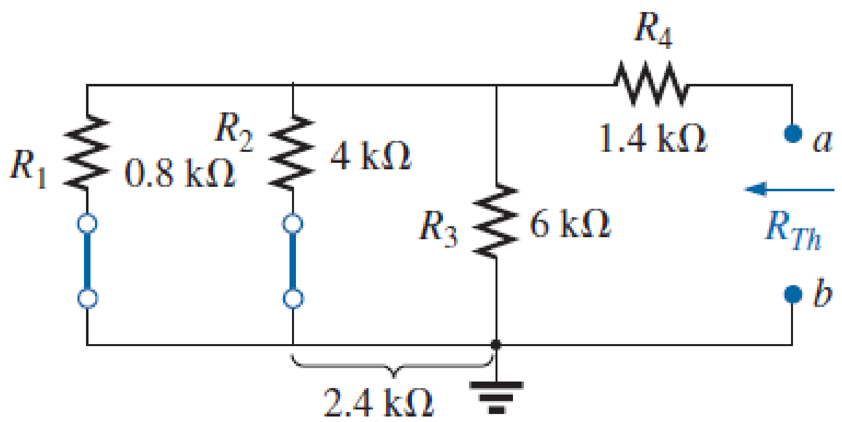


FIG. 9.50
Determining R_{Th} for the network in Fig. 9.49.

$$\begin{aligned}
R_{Th} &= R_4 + R_1 \parallel R_2 \parallel R_3 \\
&= 1.4\ \text{k}\Omega + 0.8\ \text{k}\Omega \parallel 4\ \text{k}\Omega \parallel 6\ \text{k}\Omega \\
&= 1.4\ \text{k}\Omega + 0.8\ \text{k}\Omega \parallel 2.4\ \text{k}\Omega \\
&= 1.4\ \text{k}\Omega + 0.6\ \text{k}\Omega \\
&= \mathbf{2\ k\Omega}
\end{aligned}$$

Step 4: Applying superposition, we will consider the effects of the voltage source E_1 first. Note Fig. 9.51. The open circuit requires that $V_4 = I_4R_4 = (0)R_4 = 0$ V, and

$$E'_{Th} = V_3$$

$$R'_T = R_2 \| R_3 = 4\text{ k}\Omega \| 6\text{ k}\Omega = 2.4\text{ k}\Omega$$

Applying the voltage divider rule,

$$V_3 = \frac{R'_T E_1}{R'_T} = \frac{(2.4\text{ k}\Omega)(6\text{ V})}{2.4\text{ k}\Omega + 0.8\text{ k}\Omega} = \frac{14.4\text{ V}}{3.2} = 4.5\text{ V}$$

$$E'_{Th} = V_3 = 4.5\text{ V}$$

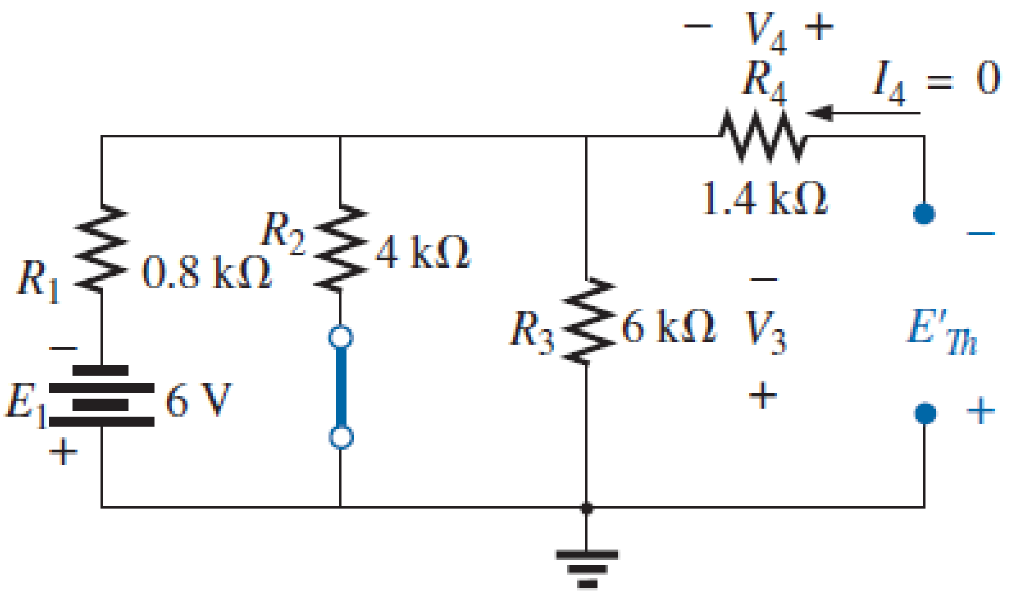


FIG. 9.51
Determining the contribution to E_{Th} from the source E_1 for the network in Fig. 9.49.

For the source E_2, the network in Fig. 9.52 results. Again, $V_4 = I_4R_4 = (0)R_4 = 0$ V, and

$$E''_{Th} = V_3$$

$$R'_T = R_1 \| R_3 = 0.8\ \text{k}\Omega \| 6\ \text{k}\Omega = 0.706\ \text{k}\Omega$$

and $$V_3 = \frac{R'_T E_2}{R'_T + R_2} = \frac{(0.706\ \text{k}\Omega)(10\ \text{V})}{0.706\ \text{k}\Omega + 4\ \text{k}\Omega} = \frac{7.06\ \text{V}}{4.706} = 1.5\ \text{V}$$

$$E''_{Th} = V_3 = 1.5\ \text{V}$$

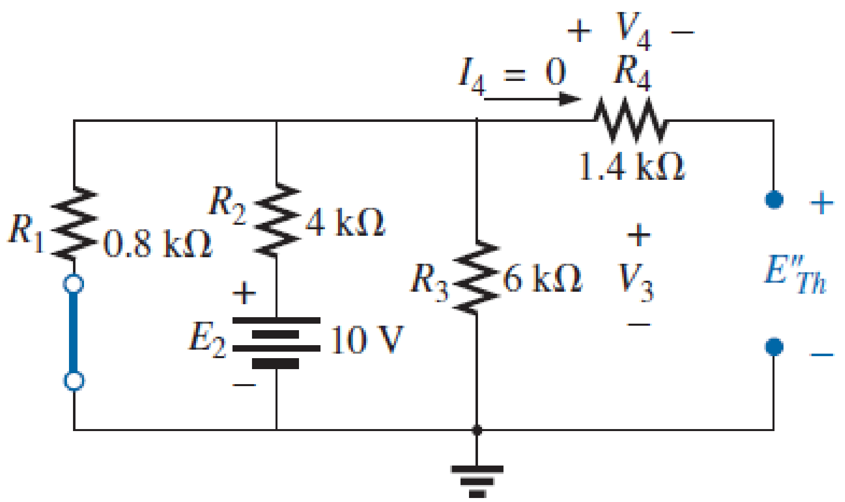


FIG. 9.52

Since E'_{Th} and E''_{Th} have opposite polarities,

$$E_{Th} = E'_{Th} - E''_{Th}$$
$$= 4.5 \text{ V} - 1.5 \text{ V}$$
$$= \mathbf{3 \text{ V}} \qquad \text{(polarity of } E'_{Th}\text{)}$$

Step 5: See Fig. 9.53.

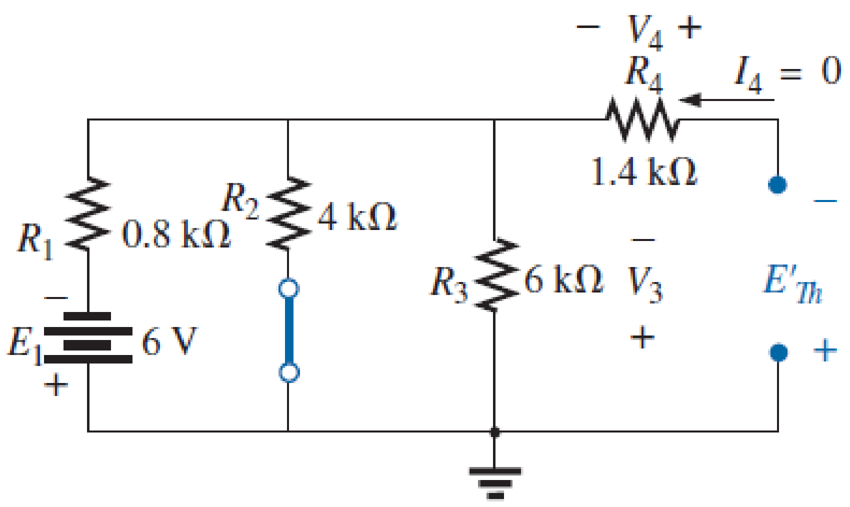


FIG. 9.51

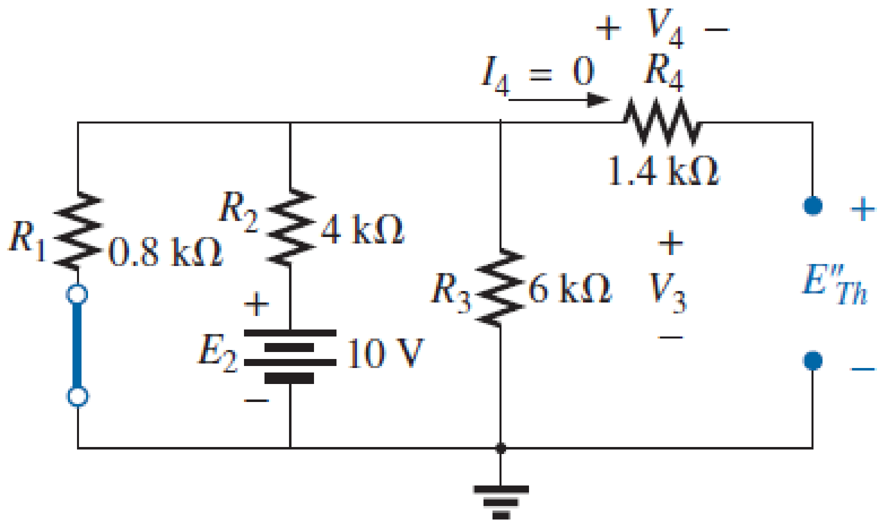


FIG. 9.52

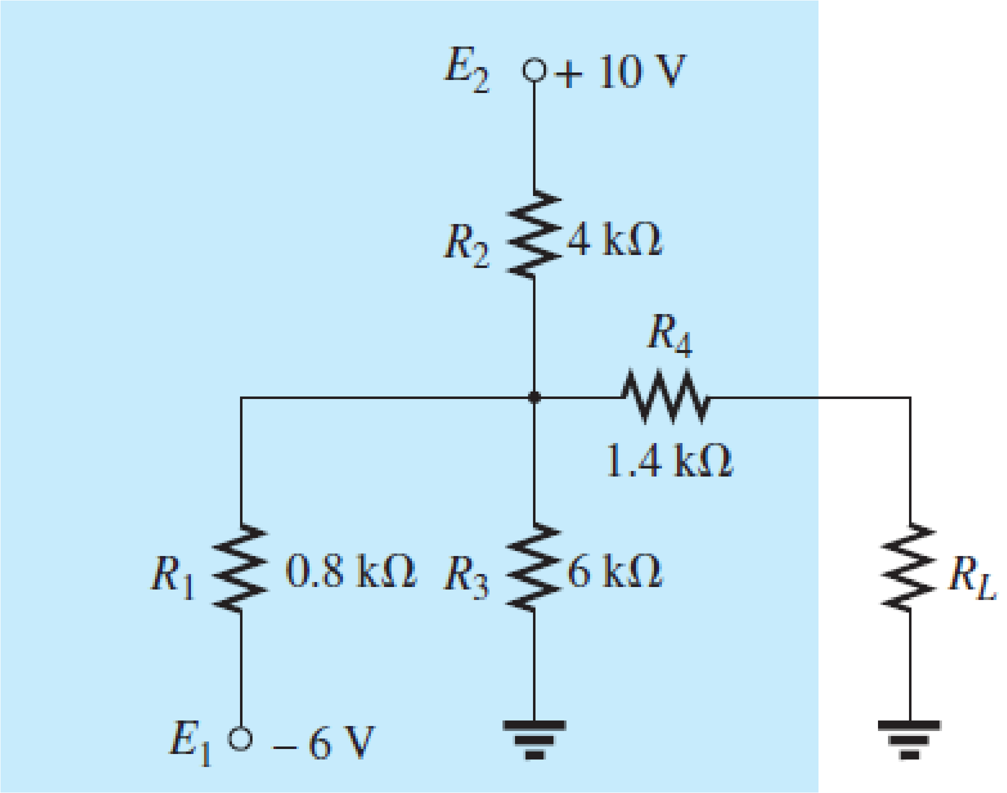


FIG. 9.48

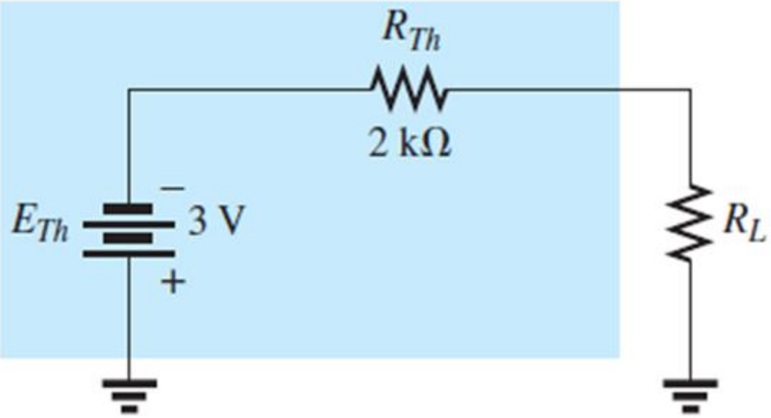


FIG. 9.53

Substituting the Thévenin equivalent circuit for the network external to the resistor R_L in Fig. 9.48.

Thank You